Assignment #16

Due on Friday, November 9, 2007

Read Section 3.1 on *The Calculus of Curves,* pp. 53–65, in Bressoud.

Read Section 5.2 on *Line Integrals,* pp. 113–119, in Bressoud.

Do the following problems

1. Consider a portion of a helix, C, parametrized by the path

$$\sigma(t) = (\cos t, t, \sin t) \quad \text{for} \quad 0 \leqslant t \leqslant \pi.$$

   Let $f(x, y, z) = x^2 + y^2 + z^2$ for all $(x, y, z) \in \mathbb{R}^3$. Evaluate $\displaystyle\int_C f$.

2. Let $f(x, y) = y$ for all $(x, y) \in \mathbb{R}^2$. For each of the following curves, C, in the plane, evaluate $\displaystyle\int_C f$.

   (a) C is the segment along the x axis from $(0, 0)$ to $(1, 0)$.

   (b) C is the segment along the y axis from $(0, 0)$ to $(0, 1)$.

   (c) C is the unit circle in $\mathbb{R}^2$.

3. Exercise 10 on page 120 in the text.

4. Exercise 12 on page 120 in the text.

5. Let f be a real valued function which is C^1 in an open interval containing the closed an bounded interval $[a, b]$. Define C to be the portion of the graph of f over $[a, b]$; that is,

$$C = \{(x, y) \in \mathbb{R}^2 \mid y = f(x),\ a \leqslant x \leqslant b\}.$$

   (a) Give a parametrization for C and compute the arc length, $\ell(C)$, of C.

   (b) Compute the arc length along the graph of $y = \ln x$ from $x = 1$ to $x = 2$.
   Note: In order to do part (b), you'll need to remember, or review, everything you learned about evaluating integrals in your single variable Calculus courses.