Assignment #18

Due on Friday, November 16, 2007

Read Section 3.1 on *The Calculus of Curves,* pp. 53–65, in Bressoud.

Read Section 5.2 on *Line Integrals,* pp. 113–119, in Bressoud.

Do the following problems

1. Exercise 4 on page 119 in the text.

2. Exercises 6(d)(e)(f) on pages 119 and 120 in the text.

3. Let $\sigma\colon [a,b] \to \mathbb{R}^n$ be a C^1 parametrization of a curve C in $\mathbb{R}^n$. Let $h\colon [c,d] \to [a,b]$ be a one–to–one and onto map such that $h'(t) \neq 0$ for all $t \in [c,d]$. Define
$$\gamma(t) = \sigma(h(t)) \quad \text{for all} \quad t \in [c,d].$$
$\gamma\colon [c,d] \to \mathbb{R}^n$ is a called a *reparametrization* of σ.

   Let $F\colon U \to \mathbb{R}^n$ denote a continuous vector field defined on a region U of $\mathbb{R}^n$ which contains the curve C. Show that
$$\int_a^b F(\sigma(\tau)) \cdot \sigma'(\tau) \, \mathrm{d}\tau = \int_c^d F(\gamma(t)) \cdot \gamma'(t) \, \mathrm{d}t.$$
   Thus, the line integral
$$\int_C F \cdot T \, \mathrm{d}s$$
   is independent of reparametrization.

4. Let $\sigma\colon [0,1] \to \mathbb{R}^n$ be a C^1 parametrization of a curve C is $\mathbb{R}^n$. Give a C^1 reparametrization, $\gamma\colon [0,1] \to \mathbb{R}^n$, of σ in which the curve C is traversed in the opposite direction as that of σ. What is $\gamma'(t)$ in terms of $\sigma'(t)$?

5. Recall that the flux of a 2–dimensional vector field,
$$F(x,y) = P(x,y)\,\widehat{\imath} + Q(x,y)\,\widehat{\jmath},$$
   across a simple, C^1, closed curve, C, is given by
$$\int_C P \, \mathrm{d}y - Q \, \mathrm{d}x.$$
   Compute the flux of the following fields across the given curves

   (a) $F(x,y) = x^2\,\widehat{\imath} + y^2\,\widehat{\jmath}$ and C is the boundary of the square with vertices $(0,0)$, $(1,0)$, $(1,1)$ and $(0,1)$.

   (b) $F(x,y) = x\,\widehat{\imath} + y\,\widehat{\jmath}$ and C is the boundary of the unit circle