Assignment #19

Due on Monday, November 19, 2007

Read Chapter 4 on *Differential Forms,* pp. 77–110, in Bressoud.

Do the following problems

1. Exercise 2 on page 86 in the text.

2. Exercises 3 and 4 on pages 86 and 87 in the text.

3. Exercises 1(b) and 1(d) on page 96 in the text.

4. Show that the directed line segment $[P_1, P_2]$ is the smallest convex set that contains the points P_1 and P_2 in $\mathbb{R}^2$; that is, if A is any convex set in $\mathbb{R}^2$ which contains the points P_1 and P_2, then
$$[P_1, P_2] \subseteq A.$$

5. Let P_1, P_2 and P_3 be three non–collinear points in $\mathbb{R}^2$. Show that the oriented triangle $T = [P_1, P_2, P_3]$ is the set
$$T = \{\alpha\overrightarrow{OP_1} + \beta\overrightarrow{OP_2} + \gamma\overrightarrow{OP_3} \mid \alpha \geqslant 0, \beta \geqslant 0, \gamma \geqslant 0, \text{ and } \alpha + \beta + \gamma = 1\},$$
where O denotes the origin in $\mathbb{R}^2$. The expression
$$\alpha\overrightarrow{OP_1} + \beta\overrightarrow{OP_2} + \gamma\overrightarrow{OP_3},$$
where α, β and γ are positive real numbers which add up to 1 is called a *convex combination* of the vectors $\overrightarrow{OP_1}$, $\overrightarrow{OP_2}$ and $\overrightarrow{OP_3}$.