Assignment #19

Due on Monday, November 19, 2007

Read Chapter 4 on *Differential Forms*, pp. 77–110, in Bressoud.

Do the following problems

1. Exercise 2 on page 86 in the text.

2. Exercises 3 and 4 on pages 86 and 87 in the text.

3. Exercises 1(b) and 1(d) on page 96 in the text.

4. Show that the directed line segment $[P_1, P_2]$ is the smallest convex set that contains the points $P_1$ and $P_2$ in $\mathbb{R}^2$; that is, if $A$ is any convex set in $\mathbb{R}^2$ which contains the points $P_1$ and $P_2$, then

$$[P_1, P_2] \subseteq A.$$

5. Let $P_1$, $P_2$ and $P_3$ be three non-collinear points in $\mathbb{R}^2$. Show that the oriented triangle $T = [P_1, P_2, P_3]$ is the set

$$T = \{ \alpha \overrightarrow{OP_1} + \beta \overrightarrow{OP_2} + \gamma \overrightarrow{OP_3} \mid \alpha \geq 0, \beta \geq 0, \gamma \geq 0, \text{ and } \alpha + \beta + \gamma = 1 \},$$

where $O$ denotes the origin in $\mathbb{R}^2$. The expression

$$\alpha \overrightarrow{OP_1} + \beta \overrightarrow{OP_2} + \gamma \overrightarrow{OP_3},$$

where $\alpha$, $\beta$ and $\gamma$ are positive real numbers which add up to 1 is called a *convex combination* of the vectors $\overrightarrow{OP_1}$, $\overrightarrow{OP_2}$ and $\overrightarrow{OP_3}$.