Assignment #20

Due on Wednesday, November 28, 2007

Read Chapter 4 on Differential Forms, pp. 77–110, in Bressoud.

Read Section 5.4 on Multiple Integrals, pp. 120–134, in Bressoud.

Do the following problems

1. Let $P$ and $Q$ denote $C^1$ scalar fields defined in some open region, $D$, or $\mathbb{R}^2$, and define the 1–form
   \[ \omega = P \, dy - Q \, dx. \]
   
   (a) Compute the differential, $d\omega$, of $\omega$.

   (b) Recall that the integral \( \int_C \omega \), where $C$ is a simple closed curve in $D$, gives the flux of the field
   \[ F = P \hat{i} + Q \hat{j} \]
   across the curve $C$.
   What does the Fundamental Theorem of Calculus,
   \[ \int_T d\omega = \int_{\partial T} \omega, \]
   where $T$ is a positively oriented triangle in $D$, say about the divergence of $F$ and its flux across the boundary of $T$?

2. Consider the iterated integral
   \[ \int_0^1 \int_y^1 e^{-x^2} \, dx \, dy. \]

   (a) Identify the region of integration, $R$, for this integral and sketch it.

   (b) Change the order of integration in the iterated integral and evaluate the double integral
   \[ \int_R e^{-x^2} \, dx \, dy. \]

3. Exercise 2 on page 135 in the text.

4. Exercise 3 on page 135 in the text.

5. Exercise 4 on page 135 in the text.