Assignment #22
Due on Monday, December 3, 2007

Read Section 8.1 on Change of Variables, pp. 211–213, in Bressoud’s book.

Background and Definitions (The Change of Variables Formula). Let $R$ denote a region in the $xy$–plane and $D$ a region in the $uv$–plane. Suppose that there is a change or coordinates function $\Phi: \mathbb{R}^2 \to \mathbb{R}^2$ that maps $D$ onto $R$. Then, for any continuous function, $f$, defined on $R$,

$$
\int_R f(x, y) \, dx\,dy = \int_D f(\Phi(u, v)) \left| \frac{\partial(x, y)}{\partial(u, v)} \right| \, dudv,
$$

where $\frac{\partial(x, y)}{\partial(u, v)}$ denotes the determinant of the Jacobian matrix of $\Phi$.

Do the following problems

1. Exercise 1 on page 216 in the text.

2. Let $D_a$ be the disc of radius $a$ in the $xy$–plane centered at the origin.

   (a) Evaluate the integral $\int_{D_a} e^{-x^2-y^2} \, dxdy$.

   (b) Compute $\lim_{a \to \infty} \int_{D_a} e^{-x^2-y^2} \, dxdy$.

3. Use your result on part (b) of the previous problem to evaluate

   $\int_{\mathbb{R}^2} e^{-x^2-y^2} \, dxdy$.

   Deduce the value of the improper integral

   $\int_{-\infty}^{+\infty} e^{-x^2} \, dx$.

4. Let $R$ be the square region in the $xy$–plane with vertices $(0, 0)$, $(1, -1)$, $(2, 0)$ and $(1, 1)$. Use the change of variables: $u = x + y$, $v = x - y$, to evaluate the integral

   $\int_{R} e^{x-y} \, dxdy$.

5. Evaluate the integral $\int_{R} 4x \, dxdy$, where $R$ is the triangular region in the $xy$–plane with vertices $(0, 0)$, $(1, 0)$ and $(1/2, 1/2)$. 