Assignment #3

Due on Wednesday September 19, 2007

Read Chapter 2 on *Vector Algebra,* pp. 29–49, in Bressoud.

Do the following problems

1. The vectors $v_1 = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$, and $\vec{v}_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ span a two–dimensional subspace in $\mathbb{R}^3$, in other words, a plane through the origin. Give two unit vectors which are orthogonal to each other, and which also span the plane.

2. Use an appropriate orthogonal projection to compute the shortest distance from the point $P(1, 1, 2)$ to the plane in $\mathbb{R}^3$ whose equation is
$$2x + 3y - z = 6.$$

3. The dual space of of $\mathbb{R}^n$, denoted $(\mathbb{R}^n)^*$, is the vector space of all linear transformations from $\mathbb{R}^n$ to $\mathbb{R}$.

   For a given $w \in \mathbb{R}^n$, define $T_w \colon \mathbb{R}^n \to \mathbb{R}$ by
$$T_w(v) = w \cdot v \quad \text{for all} \quad v \in \mathbb{R}^n.$$

   Show that T_w is an element of the dual of $\mathbb{R}^n$ for all $w \in \mathbb{R}^n$.

4. Exercise 19 on page 51 in the text.

5. Exercise 20 on page 51 in the text.