Assignment #4

Due on Friday September 21, 2007

Read Chapter 2 on *Vector Algebra,* pp. 29–49, in Bressoud.

Do the following problems

1. Let $u_1, u_2, \ldots, u_n$ be unit vectors in $\mathbb{R}^n$ which are mutually orthogonal; that is,

$$u_i \cdot u_j = 0 \quad \text{for} \quad i \neq j.$$

Prove that the set $\{u_1, u_2, \ldots, u_n\}$ is a basis for $\mathbb{R}^n$.

2. In problem 3 of Assignment #3 you were asked to show that the map $T_w \colon \mathbb{R}^n \to \mathbb{R}$ given by

$$T_w(v) = w \cdot v \quad \text{for every} \quad v \in \mathbb{R}^n$$

is an element of the dual of $\mathbb{R}^n$, denoted by $(\mathbb{R}^n)^*$; i.e., T_w is a linear, real valued function.

Prove that every linear transformation $T \colon \mathbb{R}^n \to \mathbb{R}$ must be of the form of T_w; that is, for every $T \in (\mathbb{R}^n)^*$ there exists $w \in \mathbb{R}^n$ such that

$$T(v) = w \cdot v \quad \text{for every} \quad v \in \mathbb{R}^n.$$

(*Hint:* See where T takes the standard basis $\{e_1, e_2, \ldots, e_n\}$ is $\mathbb{R}^n$.)

3. Exercises 22 and 23 on page 51 in the text.

4. In this problem and the next, we derive the vector identity

$$u \times (v \times w) = (u \cdot w)v - (u \cdot v)w$$

for any vectors u, v and w in $\mathbb{R}^3$.

   (a) Argue that $u \times (v \times w)$ lies in the span of v and w. Consequently, there exist scalars t and s such that

$$u \times (v \times w) = tv + sw$$

   (b) Show that $(u \cdot v)t + (u \cdot w)s = 0$.

5. Let u, v and w be as in the previous problem.

   (a) Use the results of the previous problem to conclude that there exists a scalar r such that

   $$u \times (v \times w) = r[(u \cdot w)v - (u \cdot v)w].$$

   (b) By considering some simple examples, deduce that $r = 1$ in the previous identity