Assignment #8

Due on Friday October 5, 2007

Read Section 7.4 on *The Derivative,* pp. 187–197, in Bressoud.

Do the following problems

1. Let f denote some real valued function defined on some open interval around $a \in \mathbb{R}$. Consider a line of slope m and equation

   $$L(x) = f(a) + m(x - a) \quad \text{for all} \;\; x \in \mathbb{R}.$$

   Suppose that this line if the best approximation to the function f at a in the sense that

   $$\lim_{x \to a} \frac{|E(x)|}{|x - a|} = 0,$$

   where $E(x) = f(x) - L(x)$ for all x in the interval in which f is defined.

   Prove that f is differentiable at a, and that $f'(a) = m$.

2. Recall that a function $F\colon U \to \mathbb{R}^m$, where U is an open subset for $\mathbb{R}^n$, is said to be differentiable at $x \in U$ if and only if there exists a unique linear transformation $T_x\colon \mathbb{R}^n \to \mathbb{R}^m$ such that

   $$\lim_{\|y - x\| \to 0} \frac{\|F(y) - F(x) - T_x(y - x)\|}{\|y - x\|} = 0.$$

   Prove that if F is differentiable at x, then it is also continuous at x.

   Give an example that shows that the converse of this assertion is not true

3. Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = \sqrt{|xy|}$ for all $(x, y) \in \mathbb{R}^2$. Show that f is not differentiable at $(0, 0)$.

4. Exercise 4 on page 197 in the text.

5. Exercise 6 on page 197 in the text.