Assignment #1

Due on Wednesday September 12, 2007

Read Chapter 2 on Vector Algebra, pp. 29–49, in Bressoud.

Do the following problems

1. Let \( \vec{v}_1 = \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} \) and \( \vec{v}_2 = \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix} \).

   (a) Give the parametric equations of the line through the point \( P: (0, 4, 7) \) in the direction of the vector \( \vec{v}_1 \).

   (b) Give the equation of the plane through the point \( P: (0, 4, 7) \) spanned by the vectors \( \vec{v}_1 \) and \( \vec{v}_2 \).

2. The following give parametric equations to two lines in \( \mathbb{R}^3 \):

   \[
   \begin{align*}
   x &= -1 + 4t \\
   y &= -7t \\
   z &= 2 - t
   \end{align*}
   \]

   \[
   \begin{align*}
   x &= -1 + s \\
   y &= 2 - s \\
   z &= 2s
   \end{align*}
   \]

   Determine if the two lines ever meet. Justify your answer. If the lines do meet, give the equation of the plane that contains both lines.

3. The following give parametric equations to two lines in \( \mathbb{R}^3 \):

   \[
   \begin{align*}
   x &= 2 + 4t \\
   y &= -1 - 7t \\
   z &= 2 - t
   \end{align*}
   \]

   \[
   \begin{align*}
   x &= s \\
   y &= 1 - s \\
   z &= -2 + 2s
   \end{align*}
   \]

   Determine if the two lines ever meet. Justify your answer. If the lines do meet, give the equation of the plane that contains both lines.

4. The vectors \( \vec{v}_1 = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} \), \( \vec{v}_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \) and \( \vec{v}_3 = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} \) in \( \mathbb{R}^3 \) can span a line, a plane or the entire three dimensional space \( \mathbb{R}^3 \). Give the equation of the geometric object which they span.

5. Exercises 10 and 11 on page 50 in the text.