Exam 2

December 7, 2007

Name: ______________________________

This is a closed book exam. Show all significant work and justify all your answers. Use your own paper and/or the paper provided by the instructor. You have 50 minutes to work on the following 3 problems. Relax.

1. Let $f\colon U \to \mathbb{R}$ be a C^1 scalar field defined on an open subset, U, of $\mathbb{R}^n$ such that $\nabla f(x) \neq \overrightarrow{0}$ for all $x \in U$. Let $\sigma\colon [a,b] \to \mathbb{R}^n$ be a C^1 path whose image is contained in U. Assume that
$$\sigma'(t) = -\nabla f(\sigma(t)) \quad \text{for all } t \in (a,b).$$
Show that the function $g(t) = f(\sigma(t))$ for all $t \in [a,b]$ is strictly decreasing on (a,b).

2. Consider the cycloid parametrized by
$$\sigma(t) = (t - \sin t, 1 - \cos t) \quad \text{for} \quad t \in \mathbb{R},$$
where t is measured in seconds.
   (a) Give the equation of the tangent line to the cycloid at the point $\left(\dfrac{3\pi}{2} + 1, 1\right)$.
   (b) Suppose a particle is moving along the cycloid and goes off on a tangent at the point $\left(\dfrac{3\pi}{2} + 1, 1\right)$. How many seconds later will the particle hit the x–axis?

3. Let C denote the boundary, ∂R, of the square, R, in xy–plane with vertices $(0,0)$, $(2,-1)$, $(3,1)$ and $(1,2)$ traversed in the counterclockwise sense.
Evaluate the following:
   (a) $\displaystyle\int_C y\ \mathrm{d}x + x\ \mathrm{d}y$.
   (b) $\displaystyle\int_R (2x - y)\ \mathrm{d}x\mathrm{d}y$.

(BONUS) Compute the arc length along the portion of the cycloid in Problem 2 from $(0,0)$ to $(2\pi, 0)$.