Assignment #9

Due on Friday, November 2, 2007

Read Chapter 4 on *Modeling bacterial growth: the continuous approach,* p. 29, in the class lecture notes webpage at `http://pages.pomona.edu/~ajr04747`

Do the following problems

1. *[The Principle of Linearized Stability].* Let $\overline{N}$ be an equilibrium point of the equation
$$\frac{dN}{dt} = g(N),$$
where g is a differentiable function with continuous derivative $g'(N)$. Show that if $g'(\overline{N}) < 0$, then $\overline{N}$ is stable, and if $g'(\overline{N}) > 0$, the $\overline{N}$ is unstable.

2. Give examples of the differential equation
$$\frac{dN}{dt} = g(N)$$
with an equilibrium point $\overline{N}$ such that $g'(\overline{N}) = 0$, and for which
   (a) $\overline{N}$ is stable, and
   (b) $\overline{N}$ is unstable.

3. Show that the initial value problem (IVP)
$$\begin{cases} \dfrac{dN}{dt} &= rN\left(1 - \dfrac{N}{K}\right) \\ N(0) &= N_o, \end{cases}$$
where $0 < N_o < K$, has solution that is defined for all real values of t. Compute
$$\lim_{t\to-\infty} N(t) \quad \text{and} \quad \lim_{t\to+\infty} N(t).$$

4. *(Population "Super–Explosion").* If the *per capita* growth is assumed to be proportional to the population density, N (that is, if

$$\frac{1}{N}\,\frac{dN}{dt} = kN$$

for some constant or proportionality $k > 0$), we obtain the model

$$\frac{dN}{dt} = kN^2. \tag{1}$$

(a) Use separation of variables to solve equation (1) subject to the initial condition $N(0) = 1$.

(b) Show that the solution obtained in part (a) ceases to exist at some (finite) time t_1. What is the value of t_1?

(c) What happens to the solution as t tends to t_1 from the left?

5. The IVP

$$\begin{cases} \dfrac{dN}{dt} &= \sqrt{N} \\ N(0) &= 0, \end{cases}$$

has the constant function 0 as a solution. Use separation of variables to compute another solution to the IVP different from the 0–solution. Why doesn't this contradict the *Local Existence and Uniqueness Theorem?*