Exam 1

October 17, 2007

Name: ______________________________

Show all significant work and justify all your answers. This is a closed book exam. Use your own paper and/or the paper provided by the instructor. You have 50 minutes to work on the following 3 problems. Relax.

1. Consider the difference equation

$$\Delta N = aN,$$

where a is a nonzero parameter.

(a) Give an interpretation of the equation as a model for population growth.

(b) Solve the equation given that N_o is known.

(c) Find equilibrium point(s) and test for stability. Which values of a yield stability?

2. The difference equation

$$\Delta N = rN(1 - N) - EN$$

provides a discrete model for a population that is growing logistically but is also being harvested at a rate proportional to the number of individuals present at a given time. The constant of proportionality E is called the *harvesting effort.* The parameters r and E are assumed to be positive.

(a) Write the model in the form $N_{t+1} = f(N_t)$ and give the fixed points of f.

(b) Find conditions on the parameters r and E that will ensure that the model will have a non–negative steady state. What does this say about the harvesting effort in terms of the intrinsic growth rate r?

(c) Use the Principle of Linearized Stability to determine conditions on r and E that will guarantee that the nonzero equilibrium point found in the previous part is stable.

(d) What does the model predict if $E = r$?

3. Figure 1 shows the graph of $N_{t+1} = f(N_t)$, where $f(x) = xe^{r(1-x)}$ for $r = 2.5$, as well as the graph of the line $N_{t+1} = N_t$.

   (a) Give the two equilibrium points of the difference equation $N_{t+1} = f(N_t)$ and determine their stability properties using the cobweb diagram.

   (b) Use the Principle of Linearized stability to verify your answers in the previous part.

   (c) Describe the long–term behavior of solutions to the difference equation whose initial values, N_o, range from 0 to 2.

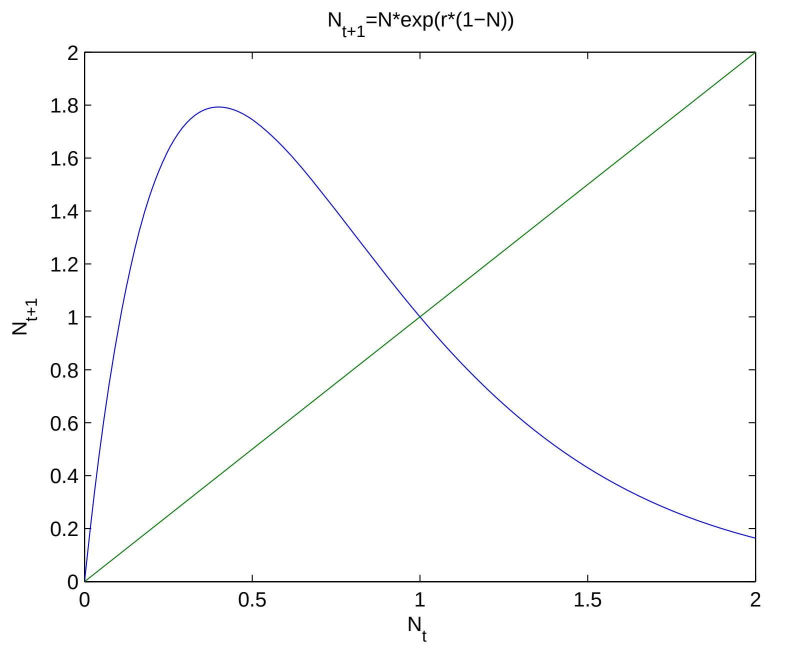

Figure 1: Cobweb diagram for $f(x) = xe^{r(1-x)}$ with $r = 2.5$