Review Problems for Exam 1

1. Find a closed form solution of the difference equation

$$X_{t+1} = \lambda X_t + a,$$

   where λ and a are real parameters, given that X_0 is known.

   Discuss how the behavior of the solution, X_t, as $t \to \infty$ is determined by the value of λ.

2. Write the difference equation of the previous problem in the form

$$X_{t+1} = f(X_t),$$

   for some function f.

   Give the equilibrium point(s) of the equation and use the principle of linearized stability to determine the nature of their stability.

3. Find the equilibrium point of the difference equation

$$X_{t+1} = \frac{X_t}{X_t + 1},$$

   and test for stability.

4. Find the equilibrium point of the difference equation

$$X_{t+1} = X_t^2 - 6,$$

   and sketch a cobweb diagram to determine their stability properties.

5. Figure 1 shows the graph of $N_{t+1} = f(N_t)$, where $f(x) = xe^{r(1-x)}$ for $r = 1.75$, as well as the graph of the line $N_{t+1} = N_t$.

   (a) Give an interpretation for the model $N_{t+1} = f(N_t)$.

   (b) In Figure 1, sketch the cobweb diagram for the solution to $N_{t+1} = f(N_t)$ satisfying $N_0 = 0.2$. What happens to the solution as $t \to \infty$?

   (c) Give the two equilibrium points of the difference equation $N_{t+1} = f(N_t)$ and determine their stability properties using the cobweb diagram.

   (d) Use the Principle of Linearized stability to verify your answers in the previous part.

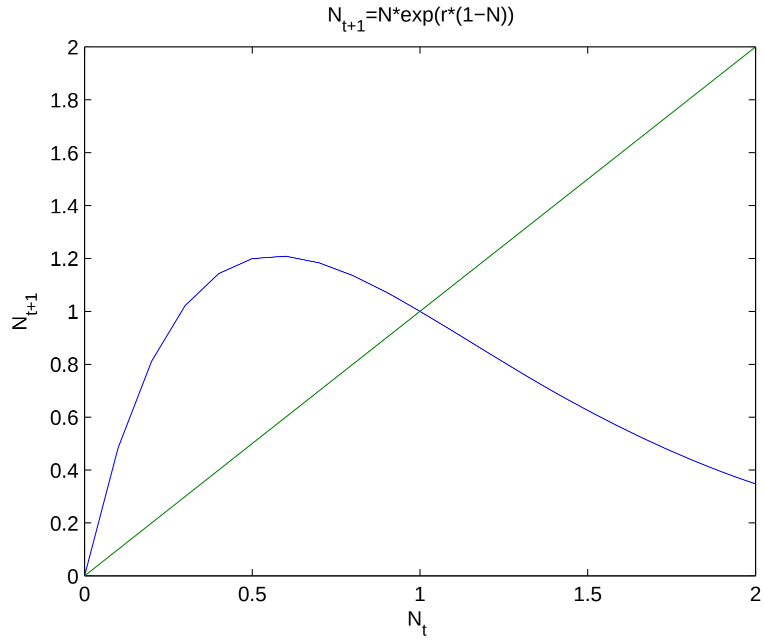

Figure 1: Cobweb diagram for $f(x) = xe^{r(1-x)}$ with $r = 1.75$

6. Investigate the following discrete model for a population of size N_t that is being harvested at a constant rate of H individuals per unit time:

$$\Delta N = rN\left(1 - \frac{N}{K}\right) - H,$$

where r, K and H are positive parameters.

7. Suppose the growth of a population of size N_t at time t is dictated by the discrete model

$$N_{t+1} = \frac{400N_t}{(10 + N_t)^2}.$$

   (a) Find the biologically reasonable fixed points for this difference equation.

   (b) Determine the stability properties of the equilibrium points found in the previous part.

   (c) If $N_0 = 5$, what happens to the population in the long run?

8. Problem 1.1.13 on page 8 in Allman and Rhodes

9. Problem 1.2.11 on pages 18–20 in Allman and Rhodes