

Assignment #14

Due on Monday, November 3, 2008

Read Section 3.1 on *The Calculus of Curves*, pp. 53–65, in Bressoud.

Read Section 5.2 on *Line Integrals*, pp. 113–119, in Bressoud.

Do the following problems

1. Let I denote an open interval in \mathbb{R} , and $\sigma: I \rightarrow \mathbb{R}^n$ be a C^1 path. For fixed $a \in I$, define

$$s(t) = \int_a^t \|\sigma'(\tau)\| \, d\tau \quad \text{for all } t \in I.$$

Show that s is differentiable and compute $s'(t)$ for all $t \in I$.

2. Let σ and s be as defined in the previous problem. Suppose, in addition, that $\sigma'(t)$ is never the zero vector for all t in I . Show that s is a strictly increasing function of t and that it is, therefore, one-to-one.
3. Let σ and s be as defined in problem (1). We can re-parameterize σ by using s as a parameter. We therefore obtain $\sigma(s)$, where s is the *arc length* parameter.

Differentiate the expression

$$\sigma(s(t)) = \sigma(t)$$

with respect to t using the Chain Rule. Conclude that, if $\sigma'(t)$ is never the zero vector for all t in I , then $\sigma'(s)$ is always a unit vector.

The vector $\sigma'(s)$ is called the *unit tangent vector* to the path σ .

4. For a and b , positive real numbers, the expression

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

defines an ellipse in the xy -plane \mathbb{R}^2 .

Sketch the ellipse, give a parametrization for it, and set up the integral that yields its arc length.

5. Let $\sigma: [0, \pi] \rightarrow \mathbb{R}^3$ be defined by $\sigma(t) = t \hat{i} + t \sin t \hat{j} + t \cos t \hat{k}$ for all $t \in [0, \pi]$. Compute the arc length of the curve parametrized by σ .