Assignment #16

Due on Friday, November 7, 2008

Read Section 3.1 on The Calculus of Curves, pp. 53–65, in Bressoud.
Read Section 5.2 on Line Integrals, pp. 113–119, in Bressoud.

Do the following problems

1. Consider a portion of a helix, $C$, parametrized by the path
   \[ \sigma(t) = (\cos t, t, \sin t) \quad \text{for} \quad 0 \leq t \leq \pi. \]
   Let \( F(x, y, z) = x \hat{i} + y \hat{j} + z \hat{k} \), for all \((x, y, z) \in \mathbb{R}^3\), be a vector field in \( \mathbb{R}^3 \). Evaluate the line integral \( \int_C F \cdot T \); that is, the integral of the tangential component of the field \( F \) along the curve \( C \).

2. Evaluate
   \[ \int_C yz \, dx + xz \, dy + xy \, dz \]
   where \( C \) is the directed line segment from the point \((1, 1, 0)\) to the point \((3, 2, 1)\) in \( \mathbb{R}^3 \).

3. Exercises 1(a)(b)(c) on page 119 in the text.

4. Exercises 1(d)(e)(f) on page 119 in the text.

5. Let \( f: U \rightarrow \mathbb{R} \) be a \( C^1 \) scalar field defined on an open subset \( U \) of \( \mathbb{R}^n \). Define the vector field \( F: U \rightarrow \mathbb{R}^n \) by \( F(x) = \nabla f(x) \) for all \( x \in U \). Suppose that \( C \) is a \( C^1 \) simple curve in \( U \) connecting the point \( x \) to the point \( y \) in \( U \). Show that
   \[ \int_C F \cdot T = f(y) - f(x). \]
   Conclude therefore that the line integral of \( F \) along a path from \( x \) to \( y \) in \( U \) is independent of the path connecting \( x \) to \( y \). The field \( F \) is called a gradient field.