Assignment #18

Due on Wednesday, November 12, 2008

Read Section 5.2 on *Line Integrals,* pp. 113–119, in Bressoud.

Background and Definitions

Consider a 2–dimensional vector field

$$F(x,y) = P(x,y)\,\widehat{\imath} + Q(x,y)\,\widehat{\jmath}.$$

If F represents a force field in two dimensions, then the line integral

$$\int_C F\cdot\mathbf{T}\ \mathrm{d}s = \int_C P\ \mathrm{d}x + Q\ \mathrm{d}y,$$

where C is the image of a C^1 path $\sigma\colon [a,b]\to\mathbb{R}^2$ which parametrizes C, represents the work done by the field in moving a particle from from $\sigma(a)$ to $\sigma(b)$ along the curve C.

If F is a two–dimensional flow field (in units of mass per unit time per unit length) and C is a C^1, simple, closed curve, then the flux of F across C,

$$\oint_C F\cdot\widehat{n}\ \mathrm{d}s = \oint_C P\ \mathrm{d}y - Q\ \mathrm{d}x,$$

gives the amount of fluid that leaves the inside of the curve C in one unit of time.

Do the following problems

1. A force field, F, is given by

$$F(x,y) = (2x+y)\,\widehat{\imath} + x\,\widehat{\jmath}.$$

   (a) Find the amount of work done by the field in moving the particle from $(1,-2)$ to $(2,1)$ along a straight line segment.

   (b) Show that the work done by the field in moving the particle from $(1,-2)$ to $(2,1)$ is independent from the path you follow to get from $(1,-2)$ to $(2,1)$.

2. A flow field, F, is given by

$$F(x,y) = 3y^2\,\widehat{\imath} - 2x\,\widehat{\jmath}.$$

Find the rate at which fluid crosses the boundary of the region in the xy–plane bounded by the curve $y = 1 - x^2$ and the line from $(-1,0)$ to $(2,-3)$.

3. Let F denote the flow field in the previous problem. Compute the flux of the field across the boundary of the triangle with vertices $(-1,0)$, $(1,0)$ and $(2,-3)$.

4. Let C be a curve parametrized by a C^1 path $\sigma\colon [a,b] \to \mathbb{R}^n$. Assume that F is perpendicular to $\sigma'(t)$ at $\sigma(t)$ for all $t \in [a,b]$. Compute the line integral of F on C.

5. Let C be a curve parametrized by a C^1 path $\sigma\colon [a,b] \to \mathbb{R}^n$. Assume that F is parallel to $\sigma'(t)$ at $\sigma(t)$ for all $t \in [a,b]$. Compute the line integral of F on C.

   Note: F is parallel to $\sigma'(t)$ at $\sigma(t)$ means that

$$F(\sigma(t)) = \lambda(t)\sigma'(t),$$

   where the scalar $\lambda(t)$ is positive.