Assignment #23

Due on Wednesday, November 26, 2008

Read Section 8.1 on *Change of Variables,* pp. 211–213, in Bressoud's book.

Background and Definitions *(The Change of Variables Formula).* Let R denote a region in the xy–plane and D a region in the uv–plane. Suppose that there is a change or coordinates function $\Phi\colon \mathbb{R}^2 \to \mathbb{R}^2$ that maps D onto R. Then, for any continuous function, f, defined on R,

$$\int_R f(x,y)\ \mathrm{d}x\mathrm{d}y = \int_D f(\Phi(u,v)) \left|\frac{\partial(x,y)}{\partial(u,v)}\right|\ \mathrm{d}u\mathrm{d}v,$$

where $\dfrac{\partial(x,y)}{\partial(u,v)}$ denotes the determinant of the Jacobian matrix of Φ.

Do the following problems

1. Exercise 1 on page 216 in the text.
2. Let D_a be the disc of radius a in the xy–plane centered at the origin.
   (a) Evaluate the integral $\displaystyle\int_{D_a} e^{-x^2-y^2}\ \mathrm{d}x\mathrm{d}y$.
   (b) Compute $\displaystyle\lim_{a\to\infty}\int_{D_a} e^{-x^2-y^2}\ \mathrm{d}x\mathrm{d}y$.
3. Use your result on part (b) of the previous problem to evaluate
   $$\int_{\mathbb{R}^2} e^{-x^2-y^2}\ \mathrm{d}x\mathrm{d}y.$$
   Deduce the value of the improper integral
   $$\int_{-\infty}^{+\infty} e^{-x^2}\ \mathrm{d}x.$$
4. Let R be the square region in the xy–plane with vertices $(0,0)$, $(1,-1)$, $(2,0)$ and $(1,1)$. Use the change of variables: $u = x+y$, $v = x-y$, to evaluate the integral
   $$\int_R e^{\,x-y}\ \mathrm{d}x\mathrm{d}y.$$
5. Evaluate the integral $\displaystyle\int_R 4x\ \mathrm{d}x\mathrm{d}y$, where R is the triangular region in the xy–plane with vertices $(0,0)$, $(1,0)$ and $(1/2,1/2)$.