Assignment #9

Due on Wednesday October 8, 2008

Read Section 7.4 on The Derivative, pp. 187–197, in Bressoud.

Do the following problems

1. Exercises 7(a) and 7(b) on pages 197 and 198 in the text.

2. Exercise 7(c) on page 198 in the text.

3. Exercise 8 on page 198 in the text.

4. Exercise 14 on pages 198 and 199 in the text.

5. A set \( U \subseteq \mathbb{R}^n \) is said to be path connected iff for any pair of vectors \( x \) and \( y \) in \( U \), there exists a differentiable path \( \sigma: [0,1] \rightarrow \mathbb{R}^n \) such that \( \sigma(0) = x \), \( \sigma(1) = y \) and \( \sigma(t) \in U \) for all \( t \in [0,1] \); that is, any two elements in \( U \) can be connected by a differentiable path whose image is entirely contained in \( U \).

   (a) Prove that the ball \( B_R(0) = \{ x \in \mathbb{R}^n \mid \| x \| < R \} \) is path connected.

   (b) Give an example in \( \mathbb{R}^2 \) of a set which is not path connected.