Exam 1 (Part II)

Due on Friday, October 17, 2008

Name: ____________________________________

This is a closed book exam. Show all significant work and justify all your answers.

4. Let $g\colon \mathbb{R} \to \mathbb{R}$ be a differentiable, real–valued function of a single variable with continuous derivative $g'(r)$ for all $r \in \mathbb{R}$. Define scalar field $f\colon \mathbb{R}^3 \to \mathbb{R}$ by

$$f(x, y, z) = g(\sqrt{x^2 + y^2 + z^2}) \quad \text{for all } (x, y, z) \in \mathbb{R}^3$$

   (a) Show that f is differentiable for all (x, y, z) in $\mathbb{R}^3$ except possibly at the origin $(0, 0, 0)$.

   Suggestion: Compute the partial derivatives of f and argue that they are continuous except possibly at the origin.

   (b) Compute the gradient of f in terms of $r = \sqrt{x^2 + y^2 + z^2}$, the derivative $g'(r)$ of g, and the vector $\overrightarrow{\mathbf{r}} = x\widehat{i} + y\widehat{j} + z\widehat{k}$.

5. Let $I \subseteq \mathbb{R}$ ba an open interval and $\sigma\colon I \to \mathbb{R}^n$ a differentiable path.

   (a) Define $g(t) = \|\sigma(t)\|^2$ for all $t \in I$. Show that g is differentiable and compute $g'(t)$ for all $t \in I$.

   (b) Suppose that $\|\sigma(t)\| = c$, a constant, for all $t \in I$. Show that $\sigma(t)$ and $\sigma'(t)$ are orthogonal (or perpendicular) to each other for all $t \in I$.