Exam 2 (Part II)

Friday, December 5, 2008

Name: ______________________________

This is a closed book exam. Show all significant work and justify all your answers. Use your own paper and/or the paper provided by the instructor. You have 50 minutes to work on the following 2 problems. Relax.

1. Let U denote an open subset in $\mathbb{R}^n$ and I an open interval on the real line. Suppose that $f\colon U \to \mathbb{R}$ is a scalar field and $\sigma\colon I \to \mathbb{R}^n$ is a path such that its image lies in U.

   (a) State the Chain Rule for the composition $f \circ \sigma$.

   (b) Suppose that f and σ are both differentiable maps such that

   $$\nabla f(x,y) \neq \overrightarrow{\mathbf{0}} \quad \text{for all } (x,y) \in U,$$

   and

   $$\sigma'(t) = -\nabla f(\sigma(t)) \quad \text{for all } t \in I.$$

   Prove that the function $f(\sigma(t))$ is strictly decreasing on I.

2. Let R denote a region in $\mathbb{R}^2$ whose boundary is made up of a finite number of C^1 pieces traversed in the counterclockwise sense.

   (a) State any of the three versions of the Fundamental Theorem of Calculus we have seen in class for the region R and an appropriately defined vector field or differential form.

   (b) Evaluate the line integral $\int_{\partial R} \omega$, where ω is the differential 1–form

   $$\omega = (x^4 + y)\,\mathrm{d}x + (2x - y^4)\,\mathrm{d}y,$$

   R is the rectangular region

   $$R = \{(x,y) \in \mathbb{R}^2 \mid -1 \leqslant x \leqslant 3,\ -2 \leqslant y \leqslant 1\},$$

   and ∂R is traversed in the counterclockwise sense.