Review Problems for Exam 2

1. For each of the following scenarios, determine whether the binomial distribution is the appropriate distribution for the random variable $X$. Explain your answer.

   (a) A fair coin is flipped ten times. Let $X$ denote the number of times the coin comes up tails.

   (b) A fair coin is flipped multiple times. Let $X$ denote the number of times the coin needs to be flipped until we see ten tails.

   (c) A roulette wheel with one ball in it is turned six times. Let $X$ denote the number of times the ball lands on red.

   (d) There are ten people in the room: five men and five women. Three people are to be selected at random to form a committee. Let $X$ denote the number of men on the three-person committee.

2. For each of the following scenarios, determine the appropriate distribution for the random variable $X$.

   (a) A fair die is rolled seven times. Let $X$ denote the number of times we see an even number.

   (b) A card is selected at random from a standard deck of shuffled cards. The color of the card is determined and the card is returned to the deck. The cards are shuffled again. This selection procedure is repeated sixty times. Let $X$ denote the proportion of times the selected card is red.

   (c) The percentage of female students at a large university is known to be 46%. A simple random sample of 100 students is to be taken. Let $X$ denote the number of male students in the sample.

   (d) On any given Saturday during college football season, there are roughly 70 games being played. At each game, a fair coin is flipped to determine which team gets to kick off first. Let $X$ denote the proportion of these coins that land heads.

3. A set of ten cards consists of five red cards and five black cards. The cards are shuffled thoroughly.

   (a) Six of these cards will be selected at random. Let $X$ denote the number of red cards observed in the set of six selected cards. Describe the probability distribution which appropriately models the random variable $X$. 

(b) One card is to be selected at random. The color will be observed and the card replaced in the set. The cards are then thoroughly reshuffled. This selection procedure is repeated four times. Let $X$ denote the number of red cards observed in these four trials. What is the mean of $X$?

4. Determine whether each of the following statements is true or false.

(a) The margin of error for a 95% confidence interval for the mean $\mu$ increases as the sample size increases.

(b) The margin of error for a confidence interval for the mean $\mu$, based on a specified sample size $n$, increases as the confidence level decreases.

(c) The margin of error for a 95% confidence interval for the mean $\mu$ decreases as the population standard deviation decreases.

(d) The sample size required to obtain a confidence interval of specified margin of error $\mu$ increases as the confidence level increases.

5. Certain Middle School has calculated a 95% confidence interval for the mean height $\mu$ of 11–year–old boys at their school and found it to be $56 \pm 2$ inches. Determine whether each of the following statements is true or false.

(a) There is a 95% probability that $\mu$ is between 54 and 58.

(b) There is a 95% probability that the true mean is 56, and there is a 95% chance that the true margin of error is 2.

(c) If we took many additional random samples of the same size and from each computed a 95% confidence interval for $\mu$, approximately 95% of these intervals would contain $\mu$.

(d) If we took many additional random samples of the same size and from each computed a 95% confidence interval for $\mu$, approximately 95% of the time $m$ would fall between 54 and 58.

6. A nationally distributed college newspaper conducts a survey among students nationwide every year. This year, responses from a simple random sample of 204 college students to the question “About how many CDs do you own?” resulted in a sample mean $\bar{X}_n = 72.8$. Based on data from previous years, the editors of the newspaper will assume that $\sigma = 7.2$.

(a) Use the information given to obtain a 95% confidence interval for the mean number of CDs owned by all college students.
(b) Answer each of the following questions with yes, no, or can’t tell.

i. Does the sample mean lie in the 95% confidence interval?

ii. Does the population mean lie in the 95% confidence interval?

iii. If we were to use a 92% confidence level, would the confidence interval from the same data produce an interval wider than the 95% confidence interval?

iv. With a smaller sample size, all other things being the same, would the 95% confidence interval be wider?

7. A review of voter registration records in a small town yielded the following table of the number of males and females registered as Democrat, Republican, or some other affiliation:

<table>
<thead>
<tr>
<th></th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>Democrat</td>
<td>300</td>
<td>600</td>
</tr>
<tr>
<td>Republican</td>
<td>500</td>
<td>300</td>
</tr>
<tr>
<td>Other</td>
<td>200</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 1: Data for Problem 7

Suppose we wish to test the null hypothesis that there is no association between party affiliation and gender. Under the null hypothesis, what is the expected number of male Democrats?

8. A specific type of electronic sensor has been experiencing failures. The manufacturer has studied 200 of these devices and has determined the type of failure (A, B, C) that occurred and the location on the sensor where the failure happened (External, Internal). The following table summarizes the findings:

<table>
<thead>
<tr>
<th>Location \ Type</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal</td>
<td>50</td>
<td>16</td>
<td>31</td>
<td>97</td>
</tr>
<tr>
<td>External</td>
<td>61</td>
<td>26</td>
<td>16</td>
<td>103</td>
</tr>
<tr>
<td>Total</td>
<td>111</td>
<td>42</td>
<td>47</td>
<td>200</td>
</tr>
</tbody>
</table>

Table 2: Data for Problem 8

Test the null hypothesis is that there is no association between Type of Failure and Location of Failure.
9. A researcher is interested in determining if the model used for the distribution of main economic concerns in the year 2003 for residents in a certain county can still be used in the year 2004. A sample of 370 residents from that county was surveyed in 2004. The following table displays the model for the distribution of economic concerns for the year 2003 and the observed number of sampled respondents in the survey for the same economic concerns for the year 2004:

<table>
<thead>
<tr>
<th></th>
<th>Jobs</th>
<th>Medical Care</th>
<th>Higher Education</th>
<th>Housing</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003 Model</td>
<td>5%</td>
<td>36%</td>
<td>27%</td>
<td>32%</td>
</tr>
<tr>
<td>2004 Counts</td>
<td>25</td>
<td>138</td>
<td>116</td>
<td>91</td>
</tr>
</tbody>
</table>

Table 3: Data for Problem 9

Perform an appropriate significance test.

10. There have been many studies that looked at the incidence of heart attack on the different days of the week. Studies in Japan and Scotland seemed to find that there was a substantial “spike” in heart attack on Mondays, perhaps as many as 20% more. A researcher studied a random selection of 200 heart attack victims and recorded the day of the week that their attack occurred. The following table summarizes the results:

<table>
<thead>
<tr>
<th>Day</th>
<th>Sunday</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counts</td>
<td>24</td>
<td>36</td>
<td>27</td>
<td>26</td>
<td>32</td>
<td>26</td>
<td>29</td>
</tr>
</tbody>
</table>

Table 4: Data for Problem 10

The researcher was interested in whether the distribution of heart attacks was the same or different across the days of the week. Perform an appropriate significance test to answer this question.