Assignment #12

Due on Monday, November 16, 2009

Read Section 3.1 on The Calculus of Curves, pp. 53–65, in Bressoud.
Read Section 5.2 on Line Integrals, pp. 113–119, in Bressoud.

Do the following problems

1. Consider a portion of a helix, $C$, parametrized by the path

$$\sigma(t) = (\cos t, t, \sin t) \quad \text{for} \quad 0 \leq t \leq \pi.$$ 

Let $f(x, y, z) = x^2 + y^2 + z^2$ for all $(x, y, z) \in \mathbb{R}^3$. Evaluate $\int_{C} f$.

2. Let $f(x, y) = y$ for all $(x, y) \in \mathbb{R}^2$. For each of the following curves, $C$, in the plane, evaluate $\int_{C} f$.

   (a) $C$ is the segment along the $x$ axis from $(0, 0)$ to $(1, 0)$.
   (b) $C$ is the segment along the $y$ axis from $(0, 0)$ to $(0, 1)$.
   (c) $C$ is the unit circle in $\mathbb{R}^2$.

3. Exercise 10 on page 120 in the text.

4. Exercise 12 on page 120 in the text.

5. Let $f$ be a real valued function which is $C^1$ in an open interval containing the closed and bounded interval $[a, b]$. Define $C$ to be the portion of the graph of $f$ over $[a, b]$; that is,

$$C = \{(x, y) \in \mathbb{R}^2 \mid y = f(x), \; a \leq x \leq b\}.$$ 

   (a) Give a parametrization for $C$ and compute the arc length, $\ell(C)$, of $C$.
   (b) Compute the arc length along the graph of $y = \ln x$ from $x = 1$ to $x = 2$.

   Note: In order to do part (b), you’ll need to remember, or review, everything you learned about evaluating integrals in your single variable Calculus courses.