Assignment #14

Due on Monday, November 23, 2009

Read Section 3.1 on *The Calculus of Curves*, pp. 53–65, in Bressoud.

Read Section 5.2 on *Line Integrals*, pp. 113–119, in Bressoud.

Do the following problems

1. Exercise 4 on page 119 in the text.

2. Exercises 6(d)(e)(f) on pages 119 and 120 in the text.

3. Let \( \sigma: [a, b] \to \mathbb{R}^n \) be a \( C^1 \) parametrization of a curve \( C \) in \( \mathbb{R}^n \). Let \( h: [c, d] \to [a, b] \) be a one–to–one and onto map such that \( h'(t) > 0 \) for all \( t \in [c, d] \). Define \( \gamma(t) = \sigma(h(t)) \) for all \( t \in [c, d] \).

\( \gamma: [c, d] \to \mathbb{R}^n \) is called a *reparametrization* of \( \sigma \).

Let \( F: U \to \mathbb{R}^n \) denote a continuous vector field defined on a region \( U \) of \( \mathbb{R}^n \) which contains the curve \( C \). Show that

\[
\int_a^b F(\sigma(\tau)) \cdot \sigma'(\tau) \, d\tau = \int_c^d F(\gamma(t)) \cdot \gamma'(t) \, dt.
\]

Thus, the line integral \( \int_C F \cdot T \, ds \) is independent of reparametrization.

4. Let \( \sigma: [0, 1] \to \mathbb{R}^n \) be a \( C^1 \) parametrization of a curve \( C \) in \( \mathbb{R}^n \). Give a \( C^1 \) reparametrization, \( \gamma: [0, 1] \to \mathbb{R}^n \), of \( \sigma \) in which the curve \( C \) is traversed in the opposite direction as that of \( \sigma \). What is \( \gamma' \) in terms of \( \sigma' \)?

5. The flux of a 2–dimensional vector field,

\[
F(x, y) = P(x, y) \hat{i} + Q(x, y) \hat{j},
\]

across a simple, closed curve, \( C \), is given by

\[
\int_C P \, dy - Q \, dx.
\]

Compute the flux of the following fields across the given curves

(a) \( F(x, y) = x^2 \hat{i} + y^2 \hat{j} \) and \( C \) is the boundary of the square with vertices \((0, 0), (1, 0), (1, 1) \) and \((0, 1)\).

(b) \( F(x, y) = x \hat{i} + y \hat{j} \) and \( C \) is the boundary of the unit disk in \( \mathbb{R}^2 \).