## Assignment #15

**Due on Monday, November 30, 2009**

**Read** Section 5.4 on *Multiple Integrals,* pp. 120–134, in Bressoud.

### Background and Definitions

- **Flux**

  Let $F = P\,\widehat{i} + Q\,\widehat{j}$, where $P$ and $Q$ are continuous scalar fields defined on an open subset, $U$, of $\mathbb{R}^2$. Suppose there is a $C^1$ simple closed curve $C$ contained in $U$. Then the flux of $F$ across $C$ is given by

  $$\int_C F\cdot\widehat{n}\;\, \mathrm{d}s = \int_C P\;\mathrm{d}y - Q\;\mathrm{d}x.$$

  Here, $\widehat{n}$ denotes a unit vector perpendicular to $C$ and pointing to the outside of $C$.

- **Divergence of a Vector Field in $\mathbb{R}^2$.**

  Given a $C^1$ vector field, $F(x,y) = P(x,y)\widehat{\mathbf{i}} + Q(x,y)\widehat{\mathbf{j}}$, defined on some open subset $U$ of $\mathbb{R}^2$, the divergence of $F$ is defined to be

  $$\mathrm{div}F(x,y) = \frac{\partial P}{\partial x}(x,y) + \frac{\partial Q}{\partial y}(x,y) \quad \text{for all } (x,y)\in U.$$

- **Green's Theorem.**

  Let $R$ denote a region in $\mathbb{R}^2$ bounded by a simple closed curve, $\partial R$, made up of a finite number of $C^1$ paths traversed in the counterclockwise sense. Let $P$ and $Q$ denote two $C^1$ scalar fields defined on some open set containing $R$ and its boundary, $\partial R$. Then,

  $$\iint_R \mathrm{div}F\;\mathrm{d}x\;\mathrm{d}y = \oint_{\partial R} F\cdot\widehat{n}\;\,\mathrm{d}s.$$

**Do** the following problems

1. Let $C$ denote the unit circle traversed in the counterclockwise direction. Evaluate the line integral $\displaystyle\int_C x^3\;\,\mathrm{d}y - y^3\,\mathrm{d}x$.

2. Let $F(x,y) = y\,\widehat{i} - x\,\widehat{j}$ and $R$ be the square in the $xy$–plane with vertices $(0,0)$, $(2,-1)$, $(3,1)$ and $(1,2)$. Evaluate $\displaystyle\int_{\partial R} F\cdot n \;\;\mathrm{d}s$.

3. Consider the iterated integral

$$\int_0^1\int_y^1 e^{-x^2}\;\;\mathrm{d}x\;\mathrm{d}y.$$

   (a) Identify the region of integration, $R$, for this integral and sketch it.

   (b) Change the order of integration in the iterated integral and evaluate the double integral

$$\int_R e^{-x^2}\;\;\mathrm{d}x\;\mathrm{d}y.$$

4. What is the region $R$ over which you integrate when evaluating the double integral

$$\int_0^1\int_{x^2}^1 x\sqrt{1-y^2}\;\;\mathrm{d}y\;\mathrm{d}x?$$

   Rewrite this as an iterated integral first with respect to $x$, then with respect to $y$. Evaluate this integral. Which order of integration is easier?

5. What is the region $R$ over which you integrate when evaluating the double integral

$$\int_1^2\int_1^x \frac{x}{\sqrt{x^2+y^2}}\;\;\mathrm{d}y\;\mathrm{d}x?$$

   Rewrite this as an iterated integral first with respect to $x$, then with respect to $y$. Evaluate this integral. Which order of integration is easier?