Assignment #8

Due on Wednesday, October 14, 2009

Read Section 7.4 on The Derivative, pp. 187–197, in Bressoud.

Do the following problems

1. Let $f$ denote a real valued function defined on some open interval around $a \in \mathbb{R}$. Consider a line of slope $m$ and equation
   
   $$L(x) = f(a) + m(x - a) \quad \text{for all } x \in \mathbb{R}.$$ 

   Suppose that this line is the best approximation to the function $f$ at $a$ in the sense that
   
   $$\lim_{x \to a} \frac{|E(x)|}{|x - a|} = 0,$$

   where $E(x) = f(x) - L(x)$ for all $x$ in the interval in which $f$ is defined. Prove that $f$ is differentiable at $a$, and that $f'(a) = m$.

2. Recall that a function $F: U \to \mathbb{R}^m$, where $U$ is an open subset for $\mathbb{R}^n$, is said to be differentiable at $u \in U$ if and only if there exists a unique linear transformation $T_u: \mathbb{R}^n \to \mathbb{R}^m$ such that
   
   $$\lim_{\|v - u\| \to 0} \frac{\|F(v) - F(u) - T_u(y - x)\|}{\|v - u\|} = 0.$$ 

   Prove that if $F$ is differentiable at $u$, then it is also continuous at $u$.

   Give an example that shows that the converse of this assertion is not true.

3. Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = \sqrt{|xy|}$ for all $(x, y) \in \mathbb{R}^2$. Show that $f$ is not differentiable at $(0, 0)$.

4. Exercise 4 on page 197 in the text.

5. Exercise 6 on page 197 in the text.