Exam 3

December 2, 2009

Name: ____________________

This is a closed book exam. Show all significant work and justify all your answers. Use your own paper and/or the paper provided by the instructor. You have 75 minutes to work on the following 5 problems. Relax.

1. Let C denote a curve in $\mathbb{R}^3$ parametrized by the path
$$\sigma(t) = (1, 3t^2, t^3), \quad \text{for } 0 \leqslant t \leqslant 1.$$
Compute the arclength, $\ell(C)$, of the curve C.

2. Let $C = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1, y \geqslant 0\}$, the upper, unit semicircle in $\mathbb{R}^2$. Compute the following path integrals:
$$\text{(a)} \quad \int_C x \, \mathrm{d}s \qquad\qquad \text{(b)} \quad \int_C y \, \mathrm{d}s$$

3. Let $F\colon \mathbb{R}^3 \to \mathbb{R}^3$ denote the vector field defined by
$$F(x, y, z) = y^2 z \, \widehat{\mathbf{i}} + 2xyz \, \widehat{\mathbf{j}} + xy^2 \, \widehat{\mathbf{k}}$$
for all $(x, y, z) \in \mathbb{R}^3$.
   (a) Find a scalar field, $f\colon \mathbb{R}^3 \to \mathbb{R}$, with the property that $F = \nabla f$.
   (b) Evaluate the line integral $\displaystyle\int_C F \cdot T \, \mathrm{d}s$, where C is the curve shown in Figure 1, on page 2 of this exam, going from the point $(-2, 4, 3)$ to the point $(3, 1, -1)$. Justify your calculations and your answer.

4. Let $F(x, y) = 2x \, \widehat{\mathbf{i}} + y \, \widehat{\mathbf{j}}$ and R be the quadrilateral region in the xy–plane with vertices $(0, 0)$, $(1, -1)$, $(2, 1)$ and $(1, 2)$. Use the divergence form of Green's Theorem to evaluate, $\displaystyle\oint_{\partial R} F \cdot n \, \mathrm{d}s$, the flux of the field F across the boundary of R.

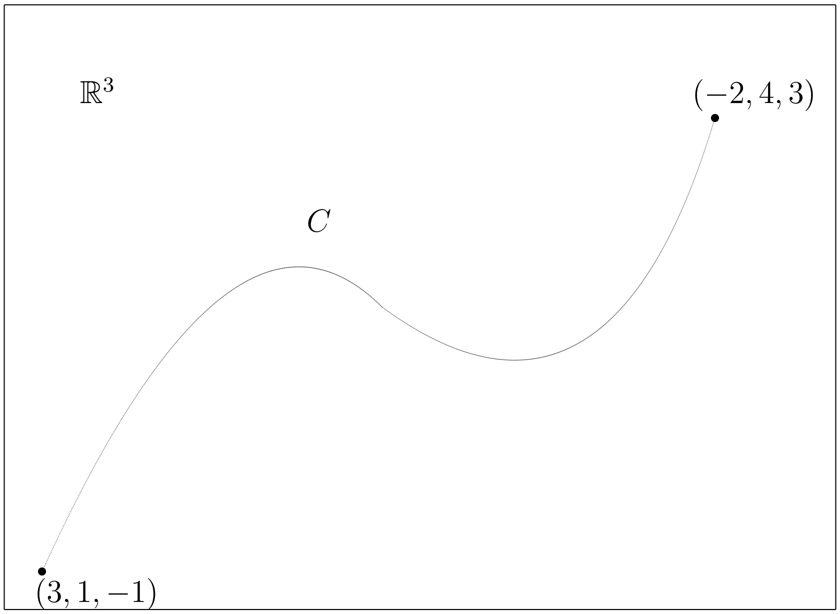

Figure 1: Curve C in part (b) of Problem 3

5. Sketch the region of integration, R, based on the iterated integral

$$\int_0^4 \int_{x/2}^2 e^{-y^2} \, \mathrm{d}y \, \mathrm{d}x,$$

and evaluate the double integral

$$\iint_R e^{-y^2} \, \mathrm{d}x \, \mathrm{d}y.$$