Exam 3

December 2, 2009

Name: ________________________________

This is a closed book exam. Show all significant work and justify all your answers. Use your own paper and/or the paper provided by the instructor. You have 75 minutes to work on the following 5 problems. Relax.

1. Let $C$ denote a curve in $\mathbb{R}^3$ parametrized by the path
   \[ \sigma(t) = (1, 3t^2, t^3), \quad \text{for } 0 \leq t \leq 1. \]
   Compute the arclength, $\ell(C)$, of the curve $C$.

2. Let $C = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1, y \geq 0\}$, the upper, unit semicircle in $\mathbb{R}^2$.
   Compute the following path integrals:
   \[ (a) \int_C x \, ds \quad (b) \int_C y \, ds \]

3. Let $F: \mathbb{R}^3 \to \mathbb{R}^3$ denote the vector field defined by
   \[ F(x, y, z) = y^2 z \, \hat{i} + 2xyz \, \hat{j} + xy^2 \, \hat{k} \]
   for all $(x, y, z) \in \mathbb{R}^3$.
   (a) Find a scalar field, $f: \mathbb{R}^3 \to \mathbb{R}$, with the property that $F = \nabla f$.
   (b) Evaluate the line integral $\int_C F \cdot T \, ds$, where $C$ is the curve shown in
       Figure 1, on page 2 of this exam, going from the point $(-2, 4, 3)$ to the
       point $(3, 1, -1)$. Justify your calculations and your answer.

4. Let $F(x, y) = 2x \, \hat{i} + y \, \hat{j}$ and $R$ be the quadrilateral region in the $xy$–plane with
   vertices $(0, 0)$, $(1, -1)$, $(2, 1)$ and $(1, 2)$. Use the divergence form of Green’s
   Theorem to evaluate, $\int_{\partial R} F \cdot n \, ds$, the flux of the field $F$ across the boundary
   of $R$. 
5. Sketch the region of integration, $R$, based on the iterated integral

$$\int_{0}^{4} \int_{x/2}^{2} e^{-y^2} \, dy \, dx,$$

and evaluate the double integral

$$\iint_{R} e^{-y^2} \, dx \, dy.$$