Topics for Exam 3

1. **Path Integrals**
   1.1 C^1 curves and parametrizations
   1.2 Arclength
   1.3 Definition of the path integral
2. **Line Integrals**
   2.1 Definition of the line integral
   2.2 Piecewise C^1 curves.
   2.3 Simple, closed curves
   2.4 Flux across a closed curve in the plane
3. **The Fundamental Theorem of Calculus**
   3.1 Gradient fields
   3.2 Divergence and flux
   3.3 Green's Theorem: Divergence form
   3.4 Double integrals

Relevant chapters and sections in the text: Section 7.4 on *The Derivative*, Section 7.6 on *The Chain Rule*, Section 3.1 on *The Calculus of Curves*, Section 5.2 on *Line Integrals* and Section 5.4 on *Multiple Integrals*.

Relevant chapters in the online class notes: Sections 4.5 and 4.6, Chapter 5, excluding sections 5.6, 5.7 and 5.9.

Important Concepts: C^1 curves, piecewise C^1 curves, simple curves, simple closed curves, parametrizations, arclength, path integral, line integral, flux, divergence and double integrals

Important Skills: Know how to evaluate the arclength of C^1 curves, know how to evaluate path integrals, know how to evaluate line integrals, know how to compute flux across a simple closed curve, know how to compute the divergence of a vector field, know how to evaluate double integrals, know how to apply the divergence form of Green's theorem.

Some Formulas

1. **Tangent Line Approximation to a C^1 Path**

   The tangent line approximation to a C^1 path $\sigma\colon [a,b] \to \mathbf{R}^n$ at $\sigma(t_o)$, for some $t_o \in (a,b)$, is the straight line given by

   $$L(t) = \sigma(t_o) + (t - t_o)\sigma'(t_o) \quad \text{for all} \quad t \in \mathbf{R}$$

2. **Arc Length**

   Let $\sigma\colon [a,b] \to \mathbf{R}^n$ be a C^1 parametrization of a curve C. The arc length of C is given by

   $$\ell(C) = \int_a^b \|\sigma'(t)\| \;\, \mathrm{d}t.$$

3. **Path Integral**

   Let $f\colon U \to \mathbf{R}$ be a continuous scalar field defined on some open subset of $\mathbf{R}^n$. Suppose there is a C^1 curve C contained in U. Then the integral of f over C is given by

   $$\int_C f \;\, \mathrm{d}s = \int_a^b f(\sigma(t))\|\sigma'(t)\| \;\, \mathrm{d}t,$$

   for any C^1 parametrization, $\sigma\colon [a,b] \to \mathbf{R}^n$ of the curve C.

4. **Line Integral**

   Let $F\colon U \to \mathbf{R}^n$ denote a continuous vector field defined on some open subset, U, of $\mathbf{R}^n$. Suppose there is a C^1 curve, C, contained in U. Then, the line integral of F over C is given by

   $$\int_C F \cdot T \;\, \mathrm{d}s = \int_a^b F(\sigma(t)) \cdot \sigma'(t) \;\, \mathrm{d}t,$$

   for any C^1 parametrization, $\sigma\colon [a,b] \to \mathbf{R}^n$, of the curve C. Here T denotes the tangent unit vector to the curve, and it is given by

   $$T(t) = \frac{1}{\|\sigma'(t)\|}\sigma'(t) \quad \text{for all} \quad t \in (a,b).$$

   If $F = P\,\widehat{i} + Q\,\widehat{j} + R\,\widehat{k}$, where P Q and R are C^1 scalar fields defined on U,

   $$\int_C F \cdot T \;\, \mathrm{d}s = \int_C P \;\, \mathrm{d}x + Q \;\, \mathrm{d}y + R \;\, \mathrm{d}z.$$

   The expression $P \;\, \mathrm{d}x + Q \;\, \mathrm{d}y + R \;\, \mathrm{d}z$ is called a differential 1–form.

5. **Flux**

   Let $F = P\,\widehat{i} + Q\,\widehat{j}$, where P and Q are continuous scalar fields defined on an open subset, U, of $\mathbf{R}^2$. Suppose there is a C^1 simple closed curve C contained in U. Then the flux of F across C is given by

$$\oint_C F \cdot \widehat{n}\ \,\mathrm{d}s = \oint_C P\ \mathrm{d}y - Q\ \mathrm{d}x.$$

   Here, $\widehat{n}$ denotes a unit vector perpendicular to C and pointing to the outside of C.

6. **Green's Theorem: Divergence Form**

   The Fundamental Theorem of Calculus in $\mathbf{R}^2$:

   Let R denote a region in $\mathbf{R}^2$ bounded by a simple closed curve, ∂R, made up of a finite number of C^1 paths traversed in the counterclockwise sense. Let P and Q denote two C^1 scalar fields defined on some open set containing R and its boundary, ∂R. Then,

$$\iint_R \mathrm{div} F\ \mathrm{d}x\ \mathrm{d}y = \oint_{\partial R} F \cdot \widehat{n}\ \,\mathrm{d}s.$$