Assignment #10

Due on Monday, October 26, 2009

Read Section 5.5 on *Introduction to Hypothesis Testing,* pp. 263–269, in Hogg, Craig and McKean.

Background and Definitions

Hypothesis Testing Terminology

- **Hypothesis Testing.** A statistical inference method that seeks to determine if a set of observations provided significant evidence to reject a hypothesis, known as a **null hypothesis** and denoted by H_o.

- **Test Statistic.** A test statistic, $T = T(X_1, X_2, \ldots, X_n)$, is a random variable based on observations, $X_1, X_2, \ldots, X_n$, and which is used to establish a criterion for rejecting H_o. The particular value of T given by a specific set of observations is denoted by $\widehat{T}$.

- p–**value.** Assuming that H_o is true, the T statistic has certain probability distribution. The distribution of T can be determined exactly, or it can be approximated. This information can then be used to compute, or approximate, the probability of observing the value of $\widehat{T}$, or more extreme values, under the assumption that H_o is true. This probability is known as the p–**value** of the test.

- **Decision Criterion.** Given certain small probability, α, known as a **significance level**, the observations provide significant evidence to reject H_o if p–value $< \alpha$. Thus,

  $$p\text{–value} < \alpha \Rightarrow \mathrm{H}_o \text{ can be rejected}$$

  at an α significance level.

- **Type I Error.** A type I error is made when a hypothesis test yields the rejection of H_o when it is in fact true. The largest probability of making a type I error is denoted by α. This is the same as the significance level of the test.

- **Type II Error**. If the null hypothesis, H_o, is in fact false, but the hypothesis test does not yield the rejection of H_o, then a type II error is made. The probability of a type II error is denoted by β.

- **Power of a Test**. The probability that H_o is rejected when it is in fact false is called the power of the test and is denoted by γ. Note the $\gamma = 1 - \beta$.

Do the following problems

1. We wish to determine whether a given coin is fair or not. Thus, we test the null hypothesis H_o: $p = \dfrac{1}{2}$ versus the alternative hypothesis H_1: $p \neq \dfrac{1}{2}$. An experiment consisting of 10 independent flips of the coin and observing the number of heads, Y, in the 10 trials. The statistic Y is the test statistic for the test. The following rejection criterion is set: Reject H_o is either $Y = 0$ or $Y = 10$.

   (a) What is the significance level of the test?

   (b) If the coin is in fact loaded, with the probability of a head being 3/4, what is the power of the test?

2. Suppose that $Y \sim \text{binomial}(100, p)$ consider a test that rejects $\text{H}_o : p = 0.5$ in favor of the alternative hypothesis $\text{H}_1 : p \neq 0.5$ provided that $|Y - 50| > 10$. Use the Central Limit Theorem to answer the following questions:

   (a) Determine α for this test.

   (b) Estimate the power of the test as a function of p and graph it.

3. Let $X_1, X_2, \ldots, X_8$ denote a random sample of size $n = 8$ from a Poissonλ distribution and put $Y = \displaystyle\sum_{j=1}^{8} X_j$. Reject the null hypothesis H_o: $\lambda = 0.5$ in favor of the alternative H_1: $\lambda > 0.5$ provided that $Y \geqslant 8$.

   (a) Compute the significance level, α, of the test.

   (b) Determine the power function, γ, as a function of λ.

4. Let $X_1, X_2, \ldots, X_n$ denote a random sample of size $n = 25$ from a Normal$\mu, 100$. Reject the null hypothesis H_o: $\mu = 0$ in favor of the alternative H_1: $\mu = 1.5$ provided that $\overline{X}_n > 0.64$.

   (a) Compute the significance level, α, of the test.

   (b) What is the power of the test?

5. Let $X_1, X_2, \ldots, X_n$ denote a random sample from a Normalμ, σ^2. Consider testing the null hypothesis

$$\text{H}_o\colon\ \mu = \mu_o \tag{1}$$

against the alternative $\text{H}_1\colon\ \mu \neq \mu_o$. Suppose that the test rejects H_o provided that $|\overline{X}_n - \mu_o| > b$ for some $b > 0$.

   (a) If the significance level of the test is α, determine the value of b. *Suggestion:* Use the $t(n-1)$ distribution.

   (b) Using the value of b found in part (b), obtain a range of values, depending on the sample mean and standard deviation, $\overline{X}_n$ and S_n, respectively, for μ_o such that the null hypothesis, H_o, in (1) is not rejected.