Assignment #6

Due on Friday, October 9, 2009

Read Section 5.3 on *More on Confidence Intervals,* pp. 254–260, in Hogg, Craig and McKean.

Do the following problems

1. Let $X_1, X_2, \ldots, X_n$ denote a random sample from a normal(μ, σ^2) distribution, where μ and σ^2 are unknown. Suppose that $n = 17$ and that the values of $X_1, X_2, \ldots, X_n$ add up to 79.90 and that the sum of the square difference from the values to the sample mean is 92.16. Give a 90% confidence interval for μ.

2. Let $X_1, X_2, \ldots, X_n$ denote a random sample from a normal(μ, σ^2) distribution, and let S_n^2 denote the sample variance. Since S_n^2 is an unbiased estimator for σ^2, $E(S_n^2) = \sigma^2$. Compute var(S_n^2); that is, compute the variance of the sampling distribution of S_n^2.

   Suggestion: Use the knowledge that you have about the distribution of $\dfrac{(n-1)}{\sigma^2} S_n^2$.

3. Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with finite variance, σ^2. Show that
$$E(S_n) \leqslant \sigma,$$
   where S_n denotes the positive square root of the sample variance.

   Furthermore, prove that $E(S_n) < \sigma$ if var$(S_n) \neq 0$.

4. Suppose we are sampling from a Bernoulli(p) distribution. Approximately, what should the sample size, n, be so that a 90% confidence interval for the parameter p has length at most 0.02.

5. Suppose we are sampling from a Poisson(λ) distribution. A sample of 200 observations from this distribution has mean equal to 3.4. Construct and approximate 90% confidence interval for λ.