Assignment #6

Due on Friday, October 9, 2009

Read Section 5.3 on More on Confidence Intervals, pp. 254–260, in Hogg, Craig and McKean.

Do the following problems

1. Let \( X_1, X_2, \ldots, X_n \) denote a random sample from a normal\((\mu, \sigma^2)\) distribution, where \( \mu \) and \( \sigma^2 \) are unknown. Suppose that \( n = 17 \) and that the values of \( X_1, X_2, \ldots, X_n \) add up to 79.90 and that the sum of the square difference from the values to the sample mean is 92.16. Give a 90% confidence interval for \( \mu \).

2. Let \( X_1, X_2, \ldots, X_n \) denote a random sample from a normal\((\mu, \sigma^2)\) distribution, and let \( S_n^2 \) denote the sample variance. Since \( S_n^2 \) is an unbiased estimator for \( \sigma^2 \), \( E(S_n^2) = \sigma^2 \). Compute \( \text{var}(S_n^2) \); that is, compute the variance of the sampling distribution of \( S_n^2 \).

   Suggestion: Use the knowledge that you have about the distribution of \( (n - 1)\frac{S_n^2}{\sigma^2} \).

3. Let \( X_1, X_2, \ldots, X_n \) be a random sample from a distribution with finite variance, \( \sigma^2 \). Show that

   \[ E(S_n) \leq \sigma, \]

   where \( S_n \) denotes the positive square root of the sample variance.

   Furthermore, prove that \( E(S_n) < \sigma \) if \( \text{var}(S_n) \neq 0 \).

4. Suppose we are sampling from a Bernoulli\((p)\) distribution. Approximately, what should the sample size, \( n \), be so that a 90% confidence interval for the parameter \( p \) has length at most 0.02.

5. Suppose we are sampling from a Poisson\((\lambda)\) distribution. A sample of 200 observations from this distribution has mean equal to 3.4. Construct and approximate 90% confidence interval for \( \lambda \).