Exam 1

September 30, 2009

Name: _____

This is a closed book exam. Show all significant work and justify all your answers. Use your own paper and/or the paper provided by the instructor. You have 50 minutes to work on the following 4 problems. Relax.

1. Define the following terms:

   (a) Random sample

   (b) Statistic

   (c) Sampling distribution

   (d) Unbiased estimator

   (e) Consistent estimator

2. Let X and Y be random variables with $X \sim \chi^2(1)$ and $Y \sim \chi^2(n)$ for $n > 1$, and define

   $$W = Y - X.$$

   Assuming that X and W are independent, determine the distribution of W.

   Suggestion: Write $Y = X + W$ and compute the mgf of Y in terms of the mgfs of X and W.

3. Let $X_1, X_2, \ldots, X_n$ be a random sample from a Poisson(λ) distribution and define the statistic $Y = \sum_{i=1}^{n} X_i$.

   (a) Derive the sampling distribution for Y. Justify your answer.

   (b) Find a value of c for that $T = cY$ is an unbiased estimator for λ. Justify your answer.

4. Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal(μ, σ^2) distribution and define the statistic

$$T_n = \sum_{i=1}^{n} (X_i - \overline{X}_n)^2,$$

where $\overline{X}_n$ denotes the sample mean. We will show later in this course that $\frac{1}{\sigma^2} T_n$ has a χ^2 distribution with $n-1$ degrees of freedom.

   (a) Explain how you would use knowledge of the distribution of $\frac{1}{\sigma^2} T_n$ to obtain a $100(1-\alpha)\%$ confidence interval for the variance σ^2 of a normal(μ, σ^2) distribution based on a random sample of size n from that distribution.

   (b) Give a 95% confidence interval for the variance of a normal(μ, σ^2) distribution based on the statistic T_n, where the sample size, n, is 17.

   (c) Assume that the counts of popcorn kernels in a 1/4 cup follow a normal distribution with parameters μ and σ^2, which are unknown. Seventeen students in this class measured a 1/4 cup of kernels and counted the kernels in the the container. The value of T_n for this particular sample of size $n = 17$ is about $21,900$. Use this information to provide a 95% confidence interval for the variance, σ^2. Give an interpretation of your result.