Exam 2

October 30, 2009

Name: ————————————————————

This is a closed book exam. Show all significant work and justify all your answers. Use your own paper and/or the paper provided by the instructor. You have 50 minutes to work on the following 5 problems. Relax.

1. Define the following terms:

    (a) Significance level of a hypothesis test.

    (b) Type II error

    (c) Power of a test

    (d) p–value

    (e) Simple hypothesis

2. Let $X_1, X_2, \ldots, X_n$ be a set of random variables with mean μ and variance σ^2, and let $\overline{X}_n = \dfrac{1}{n}\displaystyle\sum_{i=1}^{n} X_i$ and $S_n^2 = \dfrac{1}{n-1}\displaystyle\sum_{i=1}^{n}(X_i - \overline{X}_n)^2$. State the assumption(s) needed to make the following statements true.

    (a) $\dfrac{\overline{X}_n - \mu}{S_n/\sqrt{n}} \sim t(n-1)$.

    (b) $\displaystyle\sum_{i=1}^{n} \dfrac{(X_i - \mu)^2}{\sigma^2} \sim \chi^2(n)$.

3. In a simple genetic model, if both the mother and the father of a child have genotype Gg, then there is probability $1/4$ that the child will have genotype GG, probability $1/2$ of genotype Gg, and probability $1/4$ of genotype gg. In a random sample of 24 children having both parents with genotype Gg, it is found that 10 have genotype GG, 10 have genotype Gg, and 4 have genotype gg. Perform an appropriate test on the data to determine whether the data are consistent with the genetic model. Explain your reasoning.

4. A test of H_o: $\lambda = 1$ against H_1: $\lambda > 1$ is based on a test statistic, T, with a Poisson(λ) distribution. Suppose the observed value of the test statistic is $\widehat{T} = 3$.

   (a) Compute the p–value for the test.

   (b) Can H_o be rejected at the $\alpha = 0.05$ significance level?

5. Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from a normal($\mu, 1$) distribution. We wish to test the hypothesis H_o: $\mu = \mu_o$ against the alternative H_1: $\mu \neq \mu_o$. A sample of size $n = 25$ is drawn and the following rejection region is set

   $$R: \quad |\overline{X}_n - \mu_o| > c,$$

   for some critical value c.

   (a) Determine a value of c so that the significance level of the test is $\alpha = 0.05$.

   (b) Find and expression in terms of standard normal probabilities for the power function of this test.

   (c) If $\mu = \mu_o - \dfrac{1}{5}$, compute the probability of a Type II error.