## Exam 3

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This is a closed book exam. Show all significant work and justify all your answers. Use your own paper and/or the paper provided by the instructor. You have 50 minutes to work on the following 5 questions. Relax.

1. Define the following terms:

    (a) Likelihood ratio statistic

    (b) Fisher information

    (c) Efficient estimator

2. Provide concise answers to the following questions:

    (a) State the Neyman–Pearson Lemma

    (b) Give an example of an estimator which is a maximum likelihood estimator, but it is not unbiased.

    (c) State the Crámer–Rao inequality.

3. Let $X_1, X_2, \ldots, X_n$ be a random sample from a Gamma$(3, \theta)$ distribution. Find the MLE for $\theta$. Justify your answer.

4. Let $X_1, X_2, \ldots, X_n$ denote a random sample from a uniform distribution over the interval $[0, \theta]$ for some parameter $\theta > 0$ and let $W = 2\overline{X}_n$, where $\overline{X}_n$ denotes the sample mean.

    Compute the following:

    (a) $\text{bias}_\theta(W)$,
    (b) $\text{MSE}_\theta(W)$.

5. Let $X_1, X_2$ denote two independent observations from a Bernoulli$(p)$ distribution with parameter $p$, with $0 < p < 1$.

    Construct the most powerful test at a significance level $\alpha = 0.04$ to test the simple hypotheses

$$\text{H}_o: \quad p = 0.2 \quad \text{versus} \quad \text{H}_1: \quad p = 0.4.$$

    What is the power of the test?