Name: ____________________________

This is a closed book exam. Show all significant work and justify all your answers. Use your own paper and/or the paper provided by the instructor. You have 50 minutes to work on the following 5 questions. Relax.

1. Define the following terms:
   (a) Likelihood ratio statistic
   (b) Fisher information
   (c) Efficient estimator

2. Provide concise answers to the following questions:
   (a) State the Neyman–Pearson Lemma
   (b) Give an example of an estimator which is a maximum likelihood estimator, but it is not unbiased.
   (c) State the Crámer–Rao inequality.

3. Let \( X_1, X_2, \ldots, X_n \) be a random sample from a Gamma(3, \( \theta \)) distribution. Find the MLE for \( \theta \). Justify your answer.

4. Let \( X_1, X_2, \ldots, X_n \) denote a random sample from a uniform distribution over the interval \([0, \theta]\) for some parameter \( \theta > 0 \) and let \( W = 2\bar{X}_n \), where \( \bar{X}_n \) denotes the sample mean.
   Compute the following:
   (a) \( \text{bias}_p(W) \),
   (b) \( \text{MSE}_p(W) \).

5. Let \( X_1, X_2 \) denote two independent observations from a Bernoulli(\( p \)) distribution with parameter \( p \), with \( 0 < p < 1 \).
   Construct the most powerful test at a significance level \( \alpha = 0.04 \) to test the simple hypotheses
   \[ H_0: \quad p = 0.2 \quad \text{versus} \quad H_1: \quad p = 0.4. \]
   What is the power of the test?