Review Problems for Exam 2

1. In the book "Experimentation and Measurement," by W. J. Youden and published by the by the National Science Teachers Association in 1962, the author reported an experiment, performed by a high school student and a younger brother, which consisted of tossing five coins and recording the frequencies for the number of heads in the five coins. The data collected are shown in Table 1.

Number of Heads	0	1	2	3	4	5
Frequency	100	524	1080	1126	655	105

Table 1: Frequency Distribution for a Five–Coin Tossing Experiment

   (a) Are the data in Table 1 consistent with the hypothesis that all the coins were fair? Justify your answer.

   (b) Assume now that the coins have the same probability, p, of turning up heads. Estimate p and perform a goodness of fit test of the model you used to do your estimation. What do you conclude?

2. In $1,000$ tosses of a coin, 560 yield heads and 440 turn up tails. Is it reasonable to assume that the coin if fair? Justify your answer.

3. In a random sample, $X_1, X_2, \ldots, X_n$, of Bernoulli(p) random variables, it is desired to test the hypotheses $\text{H}_o\colon p = 0.49$ versus $\text{H}_1\colon p = 0.51$. Use the Central Limit Theorem to determine, approximately, the sample size, n, needed to have the probabilities of Type I error and Type II error to be both about 0.01. Explain your reasoning.

4. Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal$(\theta, 1)$ distribution. Suppose you want to test $\text{H}_o\colon \theta = \theta_o$ versus $\text{H}_1\colon \theta \neq \theta_o$, with the rejection region defined by $\sqrt{n}|\overline{X}_n - \theta_o| > c$, for some critical value c.

   (a) Find and expression in terms of standard normal probabilities for the power function of this test.

   (b) An experimenter desires a Type I error probability of 0.04 and a maximum Type II error probability of 0.25 at $\theta = \theta_o + 1$. Find the values of n and c for which these conditions can be achieved.

5. Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal(θ, σ^2) distribution. Suppose you want to test
$$\mathrm{H}_o\colon \theta \leqslant \theta_o$$
versus
$$\mathrm{H}_1\colon \theta > \theta_1$$
with the rejection region defined by
$$T_n(\theta) > \frac{\sqrt{n}}{S_n}(\theta_o - \theta) + c,$$
for some critical value c. Here, $T_n(\theta)$ is the statistic
$$T_n(\theta) = \frac{\sqrt{n}(\overline{X}_n - \theta)}{S_n},$$
where $\overline{X}_n$ and S_n^2 are the sample mean and variance, respectively.

   (a) If the significance level for the test is to be set at α, what should c be?

   (b) Express the rejection region in terms of the value c found in part (a), and the statistics $\overline{X}_n$ and S_n^2.

   (c) Compute the power function, $\gamma(\theta)$, for the test.

6. A sample of 16 "10–ounce" cereal boxes has a mean weight of 10.4 oz and a standard deviation of 0.85 oz. Perform an appropriate test to determine whether, on average, the "10–ounce" cereal boxes weigh something other than 10 ounces at the $\alpha = 0.05$ significance level. Explain your reasoning.

7. Find the p–value of observed data consisting of 7 successes in 10 Bernoulli(θ) trials in a test of
$$\mathrm{H}_o\colon \theta = \frac{1}{2} \quad \text{versus} \quad \mathrm{H}_1\colon \theta > \frac{1}{2}.$$

8. Three independent observations from a Poisson(λ) distribution yield the values $x_1 = 3$, $x_2 = 5$ and $x_3 = 1$. Explain how you would use these data to test the hypothesis $\mathrm{H}_o\colon \lambda = 1$ versus the alternative $\mathrm{H}_1\colon \lambda > 1$. Come up with an appropriate statistic and rejection criterion and determine the p–value given by the data. What do you conclude?