Topics for Final Exam

1. **Euclidean Space**

   1.1 Definition of n–dimensional Euclidean space; spans, lines and planes.

   1.2 Dot product and Euclidean norm; orthogonality and Projections

   1.3 Cross product in $\mathbf{R}^3$.

2. **Continuous Functions**

   2.1 Vector fields, scalar fields and paths

   2.2 Definition of continuous function

   2.3 Compositions of Continuous Functions

   2.4 Limits and continuity

3. **Differentiability**

   3.1 Definition of differentiability; the derivative map.

   3.2 Differentiability of paths.

   3.3 Derivatives of scalar fields: gradient; partial derivatives, directional derivatives.

   3.4 The Chain Rule

      i. Directional derivatives of scalar fields.

      ii. The Jacobian matrix of a differentiable function.

      iii. The derivative of a composition of functions.

   3.5 Divergence of a vector field.

4. **Integrals on Curves**

   4.1 Simple C^1 curves and parametrizations; re–parametrizations; piecewise C^1 simple curves; closed curves.

   4.2 Arclength of a curve; arclength parameter.

   4.3 Path integrals; line integrals; flux across a closed curve.

5. **The Fundamental Theorem of Calculus in two dimensions**

   5.1 Double integrals.

   5.2 The Divergence Theorem.

   5.3 Green's Theorem.

Relevant sections in the text: Sections 1.2, 2.1, 2.3, 2.5, 2.6, 2.7, 3.6, 3.7, 3.8, 4.1, 4.2, 4.4, 5.2, 5.3, 5.4 and 5.5.

Relevant chapters in the online class notes: Chapters 2, 3 and 4; Sections 5.1, 5.3, 5.3 and 5.4.

Important Concepts: Euclidean space; dot product; orthogonal projections; cross product; continuous function; derivative of a path; differentiability; the derivative map; partial derivatives; the gradient of a scalar field; the divergence of a vector field; C^1 curves; piecewise C^1 curves; simple curves; simple closed curves; parametrizations; re–parametrizations; arclength; path integral; line integral; flux and double integrals.

Important Skills: Know how to compute projections; know how to find equations of lines and planes; know how to show that a function is continuous or not; know how to compute the derivative of a path; know how to show whether a function is differentiable or not; know how to compute partial derivatives, gradients and directional derivative of scalar fields; know how to compute the Jacobian matrix of a differentiable map; know how to apply the Chain Rule; know how to evaluate the arclength of C^1 curves; know how to evaluate path integrals; know how to evaluate line integrals; know how to compute flux across a simple closed curve; know how to compute the divergence of a vector field; know how to evaluate double integrals; know how to apply the Fundamental Theorem of Calculus in two dimemsions.