Assignment #12

Due on Friday, October 21, 2011

Read Section 4.6 on *Integration Technique,* pp. 172–178, in *Essential Calculus with Applications* by Richard A. Silverman.

Read Section 4.7, *Linear First Order Differential Equations,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`, starting on page 54.

Do the following problems

1. Solve the initial value problem

$$\frac{dy}{dt} = -y + t, \qquad y(0) = 0.$$

2. For each $b > 0$, evaluate

$$F(b) = \int_0^b te^{-t}\, dt.$$

Then, compute $\lim_{b\to\infty} F(b)$, if it exists.

3. Let $f(t) = t \sin t$ and evaluate the area the region in the ty–plane under the graph of $y = f(t)$, bounded by the t–axis, and between $t = 0$ and $t = \pi$.

4. Let $f(t) = t \ln t$ for all $t > 0$. In Problem 3 of Assignment #5, you were asked to sketch the graph of $y = f(t)$. Evaluate the area of the region in the ty–plane which is below the t–axis and above the graph of $y = f(t)$.

5. For each $t > 0$, define $F(t)$ to be the area in the τy–pane under the graph of $y = \tau^2 e^{-\tau}$ from $\tau = 0$ to $\tau = t$.

   (a) Obtain a formula for computing $F(t)$, for $t > 0$.

   (b) Determine the values of t for which $F(t)$ increases or decreases, and the values of t for which the graph of $y = F(t)$ is concave up or concave down.

   (c) Sketch the graph of $y = F(t)$.