Assignment #14

Due on Friday, October 28, 2011

Read Section 4.9, *Solving the Logistic equation,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`, starting on page 64.

Do the following problems

1. For any population, ignoring migration, harvesting, or predation, one can model the relative growth rate by the following conservation principle

$$\frac{1}{N}\frac{dN}{dt} = \text{birth rate (per capita)} - \text{death rate (per capita)} = b - d,$$

where b and d could be functions of time and the population density N.

(a) Suppose that b and d are linear functions of N given by $b = b_o - \alpha N$ and $d = d_o + \beta N$ where b_o, d_o, α and β are positive constants. Assume that $b_o > d_o$. Sketch the graphs of b and d as functions of N. Give a possible interpretation for these graphs.

(b) Find the point where the two lines sketched in part (a) intersect. Let K denote the first coordinate of the point of intersection. Show that $K = \dfrac{b_o - d_o}{\alpha + \beta}$. K is the carrying capacity of the population.

(c) Show that $\dfrac{dN}{dt} = rN\left(1 - \dfrac{N}{K}\right)$ where $r = b_o - d_o$ is the intrinsic growth rate.

2. Assume that a population of size $N = N(t)$ grows according to a logistic model with carrying capacity of 5×10^8 individuals. Assume also that, when the population size is very small, the population doubles every 30 minutes. Suppose the initial population is 10^8. Estimate the size of the population two hours later.

3. Let $N = N(t)$ denote the size of the population described in Problem 2, where t is measured in hours. Estimate the time that it will take the population to grow to 90% of its carrying capacity.

4. Suppose that a population of size $N = N(t)$ grows according to the Logistic model. Assume that the population grows from a size N_1 to a size N_2 is an interval of time of length T. Show that

$$T = \int_{N_1}^{N_2} \frac{K}{rN(K-N)}\, dN, \tag{1}$$

where K is the carrying capacity and r is the intrinsic growth rate.

5. Suppose a population of size $N = N(t)$ grows logistically with intrinsic growth rate r and carrying capacity K. Use the formula (1) derived in Problem 4 answer the following questions.

   (a) Calculate the time that it takes for the population size to grow from $N_1 = K/4$ to $N_2 = K/2$.

   (b) What happens to T in (1) as N_2 tends to K?