Assignment #6

Due on Wednesday, September 21, 2011

Read Section 4.3 on *The Logarithm,* pp. 153–157, in *Essential Calculus with Applications* by Richard A. Silverman.

Read Section 4.4 on *The Exponential,* pp. 159–163, in *Essential Calculus with Applications* by Richard A. Silverman.

Read Section 4.5 on *More about the Logarithm and Exponential,* pp. 165–170, in *Essential Calculus with Applications* by Richard A. Silverman.

Read Section 4.2, *The Natural Logarithm Function,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`, starting on page 22.

Do the following problems

1. Show that $\displaystyle\int_1^{2.5} \frac{1}{\tau}\, d\tau < 1$ by comparing the area under the graph of $y = 1/\tau$ from $\tau = 1$ to $\tau = 2.5$ with the sum of the areas of circumscribed rectangles of width 0.25.

   Use this result to conclude that $2.5 < e$.

2. In class and in the lecture notes we showed that $2 < e < 3$. Show that
$$\int_1^{2.875} \frac{1}{\tau}\, d\tau > 1$$
by comparing the area under the graph of $y = 1/\tau$ from $\tau = 1$ to $\tau = 2.875$ with the areas of inscribed rectangles of width 0.125. Use the result of this problem and Problem 1 to conclude that $2.5 < e < 2.875$.

3. **(Base 10 Logarithm Function, or Common Logarithm)**. We say that y is the logarithm to base 10 of t if $10^y = t$. We write $y = \log t$. Thus,
$$y = \log t \quad \text{if and only if} \quad 10^y = t.$$
Solve the following equations for x using common logarithms.

   (i) $2^x = 10$; (ii) $e^x = 10$; (iii) $10^x = e$; and (iv) $b^x = a$,
   where a and b are positive real numbers

4. Suppose that $y = \log t$, for some positive real number t. Show that $y = \dfrac{\ln t}{\ln 10}$.

5. Derive the formula $\ln t = \dfrac{\log t}{\log e}$, for all $t > 0$.