Assignment #8

Due on Wednesday, September 28, 2011

Read Section 4.4 on *The Exponential,* pp. 159–163, in *Essential Calculus with Applications* by Richard A. Silverman.

Read Section 4.5 on *More about the Logarithm and Exponential,* pp. 165–170, in *Essential Calculus with Applications* by Richard A. Silverman.

Read Section 4.4, *The Exponential Function,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`, starting on page 32.

Do the following problems

1. Use the properties of ln and exp to compute the exact value of $\ln(\sqrt{e})$. Compare your result with the approximation given by a calculator.

2. Let $f(t) = te^{-t^2}$ for all $t \in \mathbf{R}$. Compute $f'(t)$ and $f''(t)$. Determine the intervals on the t–axis for which f is increasing or decreasing, and all local extrema, the values of t for which the graph of f is concave up, and those for which the graph is concave down, and all the inflection points of the graph of f. Sketch the graph of $y = f(t)$.

3. Let $f(t) = te^{-t^2}$ for all $t \in \mathbf{R}$. For each $b > 0$ compute

$$F(b) = \int_0^b te^{-t^2}\, dt;$$

that is, $F(b)$ is the area under the graph of $y = f(t)$ from $t = 0$ to $t = b$.

Compute $\lim\limits_{b\to\infty} F(b)$. We denote this limit by $\displaystyle\int_0^\infty f(t)\, dt$, and call it the improper integral of f over the interval $(0, \infty)$.

4. Define $f(t) = t^t$, for all $t > 0$, and put $g(t) = \ln[f(t)]$ for all $t > 0$.

   (a) By differentiating g with respect to t, come up with a formula for computing $f'(t)$.

   Note: You will need to apply the Chain Rule when computing $\dfrac{d}{dt}[\ln[f(t)]]$.

   (b) Compute $f''(t)$. Does the graph of $y = f(t)$ have any inflection points?

5. Let t_o, r and y_o denote real numbers. Verify that $y(t) = y_o e^{r(t-t_o)}$, for $t \in \mathbf{R}$, is the unique solutions to the initial value problem: $\dfrac{dy}{dt} = ry$, $y(t_o) = y_o$.