Assignment #9

Due on Friday, September 30, 2011

Read Section 5.2 on *Problems of Growth and Decay,* pp. 192–197, in *Essential Calculus with Applications* by Richard A. Silverman.

Read Section 4.6, *Analysis of the Malthusian Model,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`, starting on page 42.

Do the following problems

1. Assume that a certain strain of *E Coli* bacteria in a culture has a doubling time of about 30 minutes.

   (a) Assuming a Malthusian growth model for the bacteria, give an expression, $N(t)$, for the number of bacteria in the culture at time t, given that at $t = 0$ there are N_o bacteria in the culture.

   (b) How long does it take a thousand bacteria in the culture to produce one million?

2. Assume that the bacterial colony described in Problem 1 has an unlimited supply of nutrients conducive to Malthusian growth. Assume also that the bacteria are spherical with a diameter of 10^{-6} meters. Estimate the time that it would take a single bacterium of *E Coli* to grow into a mega–colony to fill the Earth's oceans, seas and bays. Use the estimate given by WolframAlpha® (http://www.wolframalpha.com/) of 1.332×10^{21} liters for the Earth's oceans, seas and bays.

3. Suppose a bacterial colony is growing according to the Malthusian model. Assume that the length of a division cycle corresponds to the doubling time. If the time, t, is measured in units of division cycle divided by $\ln 2$, give a formula for $N(t)$, given that $N(0) = N_o$. By how much does the population increase in one unit of time?

4. Assume that the rate at which a drug leaves the bloodstream and passes into the urine is proportional to the quantity of the drug in the blood at that time. Let $Q = Q(t)$ denote the amount of the drug in the bloodstream at time t. In Problem 3 of Assignment 1, you applied a conservation principle to derive the differential equation

$$\frac{dQ}{dt} = -kQ, \tag{1}$$

   where k is a positive constant of proportionality, and t is measured in hours.

(a) Solve the differential equation in (1) for the case in which an initial dose of Q_o is injected directly into the blood at time $t = 0$.

(b) Assume that 20% of the initial dose is left in the blood after 3 hours. Write a formula for computing $Q(t)$ for any time t, in hours.

(c) What percentage of the initial dosage of the drug is left in the patient's body after 6 hours?

5. In a one–compartment dilution experiment, a substance is found dissolved in water in an initial amount Q_o (in moles) in a compartment with constant volume V. Suppose pure distilled water flows into the compartment at a constant rate r (in moles per liter) and that the well–stirred mixture is drained from the tank at the same rate. Suppose that in the experiment the following concentrations of the substance were observed as a function of time:

t[sec]	C[moles/liter]
0	0.024
1	0.011
2	0.0048
3	0.0024
4	0.0010

If $Q_o = 0.1$ mole, find the flow rate r and the volume V.

(*Suggestion:* Plot the natural logarithm of the concentration, $\ln C$, versus time, t, and find the best straight line that fits the data.)