Review Problems for Exam 1

(1) Water leaks out a barrel at a rate proportional to the square root of the depth of the water at that time. If the water level starts at 36 inches and drops to 35 inches in 1 minute, how long will it take for the water to leak out of the barrel?

(2) The rate at which a drug leaves the bloodstream and passes into the urine is proportional to the quantity of the drug in the blood at that time. If an initial dose of Q_o is injected directly into the blood, 20% is left in the blood after 3 hours.

   (a) Write and solve a differential equation for the quantity, Q, of the drug in the blood at time, t, in hours.

   (b) How much of the drug is left in the patient's body after 6 hours if the patient is given 100 mg initially?

(3) Use the Fundamental Theorem of Calculus to show that $y(t) = y_o \exp(F(t))$, where F is the antiderivative of f with $F(0) = 0$, is a solution to the initial value problem $\dfrac{dy}{dt} = f(t)y, \quad y(0) = y_o$.

(4) Find a solution to the initial value problem $\dfrac{dy}{dt} = e^{t-y}, \quad y(0) = 1$.

(5) Evaluate the following integrals

$$\text{(a)} \quad \int_0^1 \frac{e^{-x}}{2 - e^{-x}} \, dx \qquad\qquad \text{(b)} \quad \int \frac{1}{x \ln x} \, dx$$

$$\text{(c)} \quad \int_1^2 \frac{\ln x}{x} \, dx \qquad\qquad \text{(d)} \quad \int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx$$

(6) The temperature in a hot iron decreases at a rate 0.11 times the difference between its present temperature and room temperature ($20°$ C).

   (a) Write a differential equation for the temperature of the iron.

   (b) If the initial temperature of the rod is $100°$ C, and the time is measured in minutes, how long will it take for the rod to reach a temperature of $25°$ C?