Exam 2

Friday, December 2, 2011

Name: _____

Show all significant work and justify all your answers. This is a closed book exam. Use your own paper and/or the paper provided by the instructor. You have 50 minutes to work on the following 3 problems. Relax.

1. In this problem you will solve the linear, first–order differential equation

$$\frac{dy}{dt} = -y + t. \tag{1}$$

   (a) Use integration by parts to evaluate the integral $\int \tau e^{\tau}\, d\tau$.

   (b) Explain why $\mu(t) = e^t$ is an integrating factor of the equation in (1).

   (c) Give the general solution to the equation in (1).

2. Consider the non–linear, first–order differential equation

$$\frac{dy}{dt} = (y-1)(y-2). \tag{2}$$

   (a) Give the equilibrium solutions to the equation in (2) and determine their stability properties. Justify your answers.

   (b) Sketch possible solutions to the differential equation in (2).

   (c) Suppose that $y = y(t)$ is a solution of (2) satisfying $y(0) = 0$. Compute $\lim_{t\to\infty} y(t)$. Justify your answer.

3. In this problem you will compute the solution to the initial value problem

$$\frac{dy}{dt} = (y-1)(y-2), \qquad y(0) = 0. \tag{3}$$

   (a) Determine constants, A and B, such that

$$\frac{1}{(y-1)(y-2)} = \frac{A}{y-1} + \frac{B}{y-2}.$$

   (b) Evaluate the integral $\int \frac{1}{(y-1)(y-2)}\, dy$.

   (c) Use separation of variable to solve the differential equation in (3) and give its general solution.

   (d) Give a formula for the solution, $y = y(t)$, to the initial value problem (3).