Solutions to Assignment #1

1. Use a Truth Table to establish the following equivalences known as one of De Morgan's laws:

   (a) $\neg(P \wedge Q) \equiv \neg P \vee \neg Q$
   Solution:

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg P \vee \neg Q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

   Observe that the columns corresponding to $\neg(P \wedge Q)$ and $\neg P \vee \neg Q$ have the same truth values. Thus, $\neg(P \wedge Q) \equiv \neg P \vee \neg Q$. $\square$

   (b) $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$
   Solution:

P	Q	$\neg P$	$\neg Q$	$P \vee Q$	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

   Observe that the columns corresponding to $\neg(P \vee Q)$ and $\neg P \wedge \neg Q$ have the same truth values. Thus, $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$. $\square$

2. Prove the following distributive properties

   (a) $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$
   Solution:

P	Q	R	$Q \vee R$	$P \wedge Q$	$(P \wedge R)$	$P \wedge (Q \vee R)$	$(P \wedge Q) \vee (P \wedge R)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

Observe that the columns corresponding to $P \wedge (Q \vee R)$ and $(P \wedge Q) \vee (P \wedge R)$ have the same truth values. Thus,

$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R).$$

$\square$

(b) $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

Solution:

P	Q	R	$Q \wedge R$	$P \vee Q$	$P \vee R$	$P \vee (Q \wedge R)$	$(P \vee Q) \wedge (P \vee R)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	F	F	F
F	F	T	F	F	T	F	F
F	F	F	F	F	F	F	F

Observe that the columns corresponding to $P \vee (Q \wedge R)$ and $(P \vee Q) \wedge (P \vee R)$ have the same truth values. Thus,

$$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R).$$

$\square$

3. Establish the following rule of reasoning known as *Modus Ponens:*

$$[(P \Rightarrow Q) \wedge P] \Rightarrow Q$$

Solution:

P	Q	$P \Rightarrow Q$	$(P \Rightarrow Q) \wedge P$	$[(P \Rightarrow Q) \wedge P] \Rightarrow Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Observe that the values in the column corresponding to $[(P \Rightarrow Q) \wedge P] \Rightarrow Q$ are always true. This establishes the result. $\square$

4. Establish the *Disjunctive Syllogism:*

$$[(P \vee Q) \wedge (\neg Q)] \Rightarrow P$$

Solution:

P	Q	$\neg Q$	$(P \vee Q)$	$[(P \vee Q) \wedge (\neg Q)]$	$[(P \vee Q) \wedge (\neg Q)] \Rightarrow P$
T	T	F	T	F	T
T	F	T	T	T	T
F	T	F	T	F	T
F	F	T	F	F	T

Observe that the truth values in the last column are always true. Thus, $[(P \vee Q) \wedge (\neg Q)] \Rightarrow P$. $\square$

5. Give the negations of the following statements.

(a) $\forall\ \varepsilon > 0\ \exists\ n \geqslant 1$ such that $\dfrac{1}{n} < \varepsilon$.

Answer: $\exists\ \varepsilon > 0$ such that $\forall\ n \geqslant 1,\ \dfrac{1}{n} \geqslant \varepsilon$ $\square$

(b) $\forall\ \varepsilon > 0\ \exists\ a \in A$ such that $a < \varepsilon$.

Answer: $\exists\ \varepsilon > 0$ such that $\forall\ a \in A,\ a \geqslant \varepsilon$ $\square$