Assignment #9

Due on Wednesday, October 31, 2012

Read Handout #2 on *The Real Numbers System Axioms.*

Read Section 4.6 on *Ordered Fields* on pp. 63–66 in Schramm's text.

Read Chapter 5 on *Upper Bounds and Suprema,* pp. 80–85, in Schramm's text.

Do the following problems

1. Let x denote a positive real number. Prove that $0 < z < 1$ implies that $zx < x$.

2. Let A and B be a non–empty subsets of $\mathbb{R}$ which are bounded from above. Prove that if $\sup A < \sup B$, then there exists $b \in B$ such that b is an upper bound for A.

3. Let A be a non–empty and bounded subset of $\mathbb{R}$. Prove that
$$\inf(A) \leqslant \sup(A).$$

4. Let $a \in \mathbb{R}$ and define the sets
$$A = \{x \in \mathbb{R} \mid x < a\}$$
and
$$B = \{q \in \mathbb{Q} \mid q < a\}.$$
Prove that the suprema of A and B exist and
$$\sup(A) = \sup(B) = a.$$

5. Use the fact that between any two distinct real numbers there is a rational number to prove the statement:

   Between any two distinct real numbers there is at least one irrational number.