Exam 1 (Part II)

Friday, October 12, 2012

Name: ______________________________

This is the out–of–class portion of Exam 1. There is no time limit for working on the following three problems. You are only allowed to consult Handout #2 on the axioms of the real numbers.

Provide complete arguments when asked to prove a statement in a question. You will be graded on how well you organize your proofs as well as the logical flow or your deductions.

Write your name on this page and staple it to your solutions.

Due on Monday, October 15, 2012

1. Let A be a non–empty subset of $\mathbb{R}$. Prove that if u is an upper bound for A and $u \in A$, then $u = \sup A$.

2. In each of the following, show that the given set A is bounded, and compute $\sup(A)$ and $\inf(A)$.

   (a) $A = \{x \in \mathbb{R} \mid 0 < x < 1\}$; in other words, A is the open interval $(0, 1)$.

   (b) $A = \left\{ \dfrac{n}{n+1} \;\middle|\; n \in \mathbb{N} \right\}$.

3. Let $B \subseteq \mathbb{R}$ be a non–empty subset which is bounded from below and put $\ell = \inf B$. Prove that for every $n \in \mathbb{N}$ there exists $x_n \in B$ such that
$$\ell \leqslant x_n < \ell + \frac{1}{n}.$$