Assignment #1

Due on Wednesday, September 12, 2012

Read Chapter 2, *Introductory Example: Recovering a Function from its Rate of Change,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Do the following problems

1. Experiments designed to estimate the evaporation rate of water in swimming pools have shown that, for an outdoor swimming pool with wind speed of 0.06 mph and temperature of 84°F, the evaporation rate for each square foot of water surface is about 0.07 pounds per hour. Consider an outdoor swimming pool which has dimensions 25 meters by 12.5 meters, with and average depth of 1.40 meters.

   (a) Assume that the conditions of temperature and wind speed mentioned above hold throughout the day. Estimate how much water evaporates from the pool in a day.

   (b) Suppose that you start with a full pool on a given day. Call that time $t = 0$. Let $Q(t)$ denote the volume of water in the pool on day t. What is the initial volume $Q_o = Q(0)$ of water in the swimming pool? (Water has a density of about 1.00 gram per cubic centimeter.)

   (c) Assume that the water evaporates from the pool at a constant rate and that water lost through evaporation is not replenished. Give a formula for computing the volume of water reamining in the pool, $Q(t)$, where t is the number of days after the pool was first filled completely.

   (d) How many days will it be until the pool contains half of its original volume? How long will it take for all the water to evaporate from the pool?

2. Rivers presently carry about 10^{10}m^3 of soil and rock to the sea each year throughout the world. Roughly how long will it take at that rate for the continents to shrink by an average of 1 meter in elevation if all other processes (such as continental uplift) are ignored? You may use the fact that the total area of the continents is about $1.48 \times 10^{14}\text{m}^2$ and that their mean elevation is about 840 meters. Assume also that the total area of the continents remains constant.

3. In 2010 the world consumption rate of crude oil was estimated to be $86,952.47$ thousand barrels a day. As of December 2003, the Oil & Gas Journal reported the world wide crude oil reserves to be 1.28 trillion barrels. Assume that we continue to consume petroleum at the 2010 consumption rate, in what year will we use up the estimated worldwide resource of this fuel?

4. For a positive integer n, define the sum
$$S_n = \sum_{k=1}^{n} k = 1 + 2 + \cdots + (n-1) + n. \tag{1}$$
Re–write the sum in (1) as
$$S_n = n + (n-1) + \cdots + 2 + 1. \tag{2}$$
Add the two expressions in (1) and (2) to get that
$$2S_n = n(n+1). \tag{3}$$
Explain your reasoning.

Use the result in (3) to derive the formula
$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$$
Explain your reasoning.

5. In this problem we obtain a formula for computing $\displaystyle\sum_{k=1}^{n} k^2$.

   (a) Write out the sum $\displaystyle\sum_{k=1}^{n}[(k+1)^3 - k^3]$ to see that you obtain $(n+1)^3 - 1$. Explain your reasoning.

   (b) Verify that $(k+1)^3 - k^3 = 3k^2 + 3k + 1$.

   (c) Combine the results in parts (a) and (b) above to conclude that
$$3\sum_{k=1}^{n} k^2 + 3\sum_{k=1}^{n} k + n = (n+1)^3 - 1. \tag{4}$$
Explain your reasoning.

   (d) Use the result from Problem 4 and the equation in (4) to derive
$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$
Explain your reasoning.