Assignment #10

Due on Wednesday, October 31, 2012

Read Section 5.3, *The Area Function as a Riemann Integral,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 15-5, pp. 322–324, in *The Calculus Primer* by William L. Schaaf.

Background and Definitions

- **Theorem** *(Some Integration Facts).* Let C denote an arbitrary constant.

  (i) $\displaystyle\int k\ dx = kx + C$, for any constant k.

  (ii) $\displaystyle\int x^m\ dx = \frac{1}{m+1}x^{m+1} + C$ for $m = 1, 2, 3, \ldots$.

  (iii) $\displaystyle\int \cos x\ dx = \sin x + C$.

Do the following problems

1. *Translate of a Function.* Let f denote a piecewise continuous function that is defined for all real numbers.

   For any given constant, c, define another function, denoted f_c, by

   $$f_c(t) = f(t - c), \qquad \text{for all } t \in \mathbb{R}.$$

   (a) What is f_0?

   (b) If $c > 0$. Explain why the graph of f_c is the same as that of f translated a distance c to the right.

2. *Translate of a Function (continued).* Let f and c be as in Problem 2. Explain why the integration formula

   $$\int_c^{c+x} f_c(t)\ dt = \int_0^x f(t)\ dt, \qquad \text{for all } x \in \mathbb{R}, \tag{1}$$

   is true.

3. In the lecture notes we derived the integration formula

$$\int_a^x \cos t\ dt = \sin x - \sin a, \qquad \text{for } x \in \mathbb{R} \text{ and } a \in \mathbb{R}.$$

Use the integration formula in (1) and the trigonometric identities

$$\begin{aligned} \sin(\alpha - \beta) &= \sin\alpha\cos\beta - \cos\alpha\sin\beta \\ \sin(\alpha + \beta) &= \sin\alpha\cos\beta + \cos\alpha\sin\beta \end{aligned}$$

to derive a formula for computing $\displaystyle\int_0^x \sin t\ dt$.

Evaluate the indefinite integral $\displaystyle\int \sin x\ dx$.

4. *Area Between the Graphs of Two Functions.* Let f and g denote two piecewise continuous functions defined on the interval $[a, b]$. Suppose that

$$f(t) \leqslant g(t), \qquad \text{for all } t \in [a, b].$$

Let R denote the region in the ty–plane that lies below the graph of g, above the graph of f, and between the lines $t = a$ and $t = b$. Explain why

$$\text{area}(R) = \int_a^b [g(t) - f(t)]\ dt.$$

5. Let $f(t) = 2t$ and $g(t) = 3 - t^2$ for all $t \in \mathbb{R}$. Compute the area of the region bounded by the graphs of f and g over the interval $[-3, 1]$.