Assignment #11

Due on Friday, November 2, 2012

Read Section 5.3, *The Area Function as a Riemann Integral,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 15-5, pp. 322–324, in *The Calculus Primer* by William L. Schaaf.

Do the following problems

1. *(A Geometric Sum).* For a given real number, r, define the sum

$$S_n = \sum_{k=1}^{n} r^k = r + r^2 + r^3 + \cdots + r^n.$$

   (a) Multiply S_n by r and subtract S_n from rS_n to obtain the expression

$$(r-1)S_n = r^{n+1} - r.$$

   (b) Assume that $r \neq 1$ and derive the formula

$$\sum_{k=1}^{n} r^k = \frac{r^{n+1} - r}{r-1}, \qquad \text{for } r \neq 1,$$

2. *(Estimating Logarithms).*

   (a) Refer to the sketch in Figure 1.

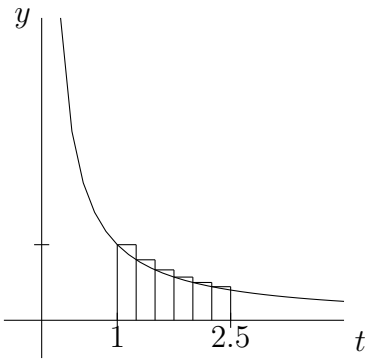

Figure 1: Sketch of graph of $y = 1/t$ with circumscribed rectangles

Find an upper estimate for $\ln(2.5)$ by comparing $\displaystyle\int_1^{2.5} \frac{1}{t}\, dt$ with the area of the circumscribed rectangles shown in the figure.

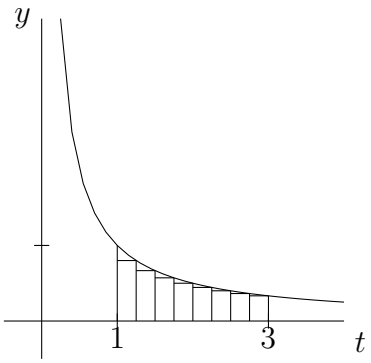

Figure 2: Sketch of graph of $y = 1/t$ with inscribed rectangles

(b) Refer to the sketch in Figure 2.

Find a lower estimate for ln(3) by comparing $\displaystyle\int_1^3 \frac{1}{t}\,dt$ with the area of the inscribed rectangles shown in the figure.

3. Compute the area bounded by the graphs of $y = \cos t$ and $y = \sin t$ over the interval $[0, 2\pi]$

4. Compute the area bounded by the graphs of $y = t^2 - 4$ and the t–axis over the interval $[-3, 3]$

5. Let $f(t) = \dfrac{1}{t^2}$, for all $t > 0$. For each natural number n, define

$$a_n = \int_1^n f(t)\,dt$$

(a) Compute a_n for all $n \in \mathbb{N}$.

(b) Determine whether or not the sequence (a_n) converges. If (a_n) converges, compute $\lim_{n\to\infty} a_n$.