Assignment #13

Due on Monday, November 19, 2012

Read Section 6.1, *Instantaneous Rate of Change,* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read Sections 1–7, 1–8 and 1–9, pp. 27–32, in *The Calculus Primer* by William L. Schaaf.

Read Sections 2–1, 2–2, 2–3, 2–4 and 2–5, pp. 47–54, in *The Calculus Primer* by William L. Schaaf.

Background and Definitions

- *(The Derivative of a Function).* Let f be a function defined on an open interval I and $t \in I$. If the limit
$$\lim_{h\to 0} \frac{f(t+h) - f(t)}{h} \tag{1}$$
exists, we call it the **instantaneous rate of change** of f at t. If the limit in (1) exists, we denote it by $f'(t)$, and call $f'(t)$ the **derivative** of f at t. We then have that
$$f'(t) = \lim_{h\to 0} \frac{f(t+h) - f(t)}{h}, \tag{2}$$
provided that the limit in (1) exists.

- *(Difference Quotient).* The expression $\dfrac{f(t+h) - f(t)}{h}$, for $h \neq 0$, is called the difference quotient of f from t to $t+h$, and is denoted by $\dfrac{\Delta f}{\Delta t}$, read, "the change in f over the change in t." Thus, according to (2), if the limit in (1) exists,
$$f'(t) = \lim_{\Delta t\to 0} \frac{\Delta f}{\Delta t}. \tag{3}$$

- *(Differential Notation).* If the limit on the right–hand side of (3) exists, we denote it by $\dfrac{df}{dt}$. We then have that $f'(t) = \dfrac{df}{dt}$. The symbol df is called the differential of f and dt is the differential of t.

Do the following problems

1. Let $f(t) = t^{1/3}$ for all $t \in \mathbb{R}$. Show that the instantaneous rate of of f at 0 does not exist.

2. Let $f(t) = t^{1/3}$ for all $t \in \mathbb{R}$.

   (a) Use the factorization fact

   $$a^3 - b^3 = (a-b)(a^2 + ab + b^2$$

   to derive the identity

   $$h = [(t+h)^{1/3} - t^{1/3}][(t+h)^{2/3} + t^{1/3}(t+h)^{1/3} + t^{2/3}]. \qquad (4)$$

   (b) Use the identity in (4) to show that, for $t \neq 0$, the limit

   $$\lim_{h \to 0} \frac{(t+h)^{1/3} - t^{1/3}}{h}$$

   exists, and compute $f'(t)$ for $t \neq 0$.

3. Let $f(t) = c$ for all $t \in \mathbb{R}$, where c is a constant. Show that the instantaneous rate of change of f exists for all t and compute $\dfrac{dc}{dt}$, for all t.

4. Let $f(t) = t$ for all $t \in \mathbb{R}$. Show that $f'(t)$ exists for all t and compute $\dfrac{dt}{dt}$, for all t.

5. Let $f(t) = t^2$ for all $t \in \mathbb{R}$. Show that $f'(t)$ exists for all t and compute $\dfrac{df}{dt}$, for all t.