Assignment #17

Due on Wednesday, December 5, 2012

Read Chapter 7, *Fundamental Theorems,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 12–1, pp. 252–254, in *The Calculus Primer* by William L. Schaaf.

Do the following problems

1. Let $f(x) = \displaystyle\int_0^x \frac{1}{1+t^4}\,dt$, for all $x \in \mathbb{R}$.

   Explain why f is differentiable in $\mathbb{R}$ and compute f'.

2. Let $F(x) = \displaystyle\int_0^x \frac{1}{\sqrt{1-t^2}}\,\mathrm{d}t$ for $-1 < x < 1$.

   (a) Use the Second Fundamental Theorem of Calculus in the notes to compute $F'(x)$ for $-1 < x < 1$.

   (b) Determine the values of x in $(-1, 1)$ at which $F(x)$ increases with x, and those at which it decreases.

3. Let $f(t) = \begin{cases} -1, & \text{if } t < 0; \\ t, & \text{if } t \geqslant 0, \end{cases}$ and define $F(x) = \displaystyle\int_{-2}^x f(t)\,dt$, for all $x \in \mathbb{R}$.

   (a) Sketch the graph of $y = f(t)$.

   (b) Sketch the graph of $y = F(x)$.

   (c) Show that F is not differentiable at 0. Explain why this does not contradict the Second Fundamental Theorem of Calculus in the notes.

4. Let g be differentiable over some interval, I, and suppose that $g(t) > 0$ for all $t \in I$. Define

$$f(t) = \ln[g(t)], \quad \text{for all } t \in I.$$

   (a) Explain why f is differentiable and compute f'.

   (b) Use your result from part (a) to obtain an integration formula for

$$\int \frac{g'(x)}{g(x)}\,dx.$$

5. Find a continuous function, f, defined on positive real values, and a number $a > 0$ such that

$$6 + \int_a^x \frac{f(t)}{t^2}\,\mathrm{d}t = 2\sqrt{x} \quad \text{for all } x > 0.$$