**Assignment #5**

**Due on Wednesday, September 26, 2012**

**Read** Section 3.2, *Limits of Functions,* in the class lecture notes at
http://pages.pomona.edu/~ajr04747/

**Read** on *The Limit Concept,* pp. 32–45, in *The Calculus Primer* by William L. Schaaf.

**Background and Definitions**

*The Squeeze Lemma.* Let $f$, $g$ and $h$ denote a functions whose domains consist of union of intervals that either contain $a$, or $a$ is an end–point of some of the intervals. (Note that $a$ might or might not be in the domains of $f$, $g$ or $h$). Suppose that there exists a positive number $\delta$ such that

$$f(t) \leqslant g(t) \leqslant h(t), \quad \text{for } |t-a| < \delta,$$

and $t$ is in the domains of $f$, $g$ and $h$. Assume in addition that the limits of $f$ and $h$ as $t$ approaches $a$ exist and that $\lim_{t\to a} f(t) = \lim_{t\to a} h(t) = L$. Then, the limit of $g$ as $t$ approaches $a$ exists and $\lim_{t\to a} g(t) = L$.

**Do** the following problems

1. Refer to the sketch of the unit circle in Figure 1. Derive the inequalities

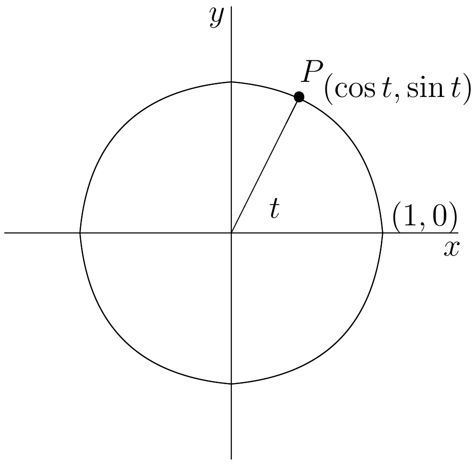


Figure 1: Unit Circle

$$-1 \leqslant \cos t \leqslant 1 \quad \text{and} \quad -1 \leqslant \sin t \leqslant 1, \quad \text{for all } t.$$

2. Refer to the sketch of the unit circle in Figure 1.

   (a) Derive the inequality
   $$|\sin t| \leqslant |t|, \quad \text{for } |t| < \frac{\pi}{2}. \tag{1}$$

   (b) Use the inequality in (1) and the Squeeze Lemma to show that
   $$\lim_{t\to 0} \sin t = 0. \tag{2}$$

3. Use the trigonometric identities
   $$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \tag{3}$$
   and
   $$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta \tag{4}$$
   to derive the identity
   $$\cos(\alpha+\beta) - \cos(\alpha-\beta) = -2\sin\alpha\sin\beta. \tag{5}$$
   *Suggestion:* Subtract the equation in (4) from that in (3).

4. Use the identity in (5) to derive the identity
   $$\cos(t) - \cos(a) = -2\sin\left(\frac{t+a}{2}\right)\sin\left(\frac{t-a}{2}\right). \tag{6}$$
   *Suggestion:* Set $t = \alpha+\beta$, $a = \alpha-\beta$, and solve for $\alpha$ and $\beta$ in terms of $t$ and $a$.

5. *Computing* $\lim_{t\to a} \cos t$.

   (a) Use the identity in (6) and the results in Problems 1 and 2 to derive the inequality
   $$|\cos(t) - \cos(a)| \leqslant |t-a|, \quad \text{for } |t-a| < \pi. \tag{7}$$

   (b) Use (7) and the Squeeze Lemma to deduce that
   $$\lim_{t\to a} \cos t = \cos a, \quad \text{for all } a.$$